Stability of Networked Control Systems with Random Buffer Capacity

Dongxiao Wu[1], Jun Wu[1], Sheng Chen[2]

[1] College of Information Science and Engineering
Zhejiang University

[2] School of Electronics and Computer Science
University of Southampton

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Outline

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Motivation
- Networked Control Systems(NCSs)
- Existing Works & Our Approach

Main Result
- Problem Formulation
- Stability Analysis

Example

Some Basics

In NCS, the plant and the controller exchange data via a shared communication network.

Advantages:

- Low installation cost.
- Reduced system wiring.
- Easy maintenance.

Problems:

- Bandwidth constraint.
- Packet delay.
- Packet dropout.

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Highlights

- Existing works:
  1. Only use the most recent sensor value.
  2. Sufficient large buffer capacity.
- Our approach:
  1. Use current and past sensor values.
  2. Buffer capacity constraint.

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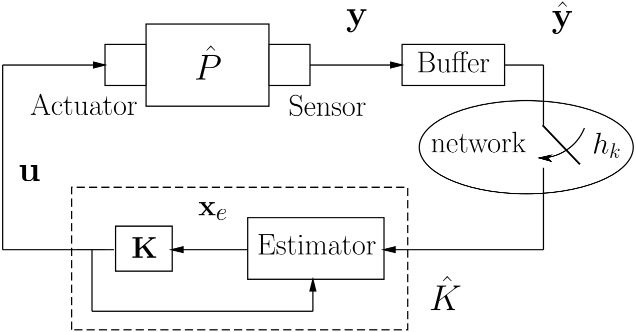
Networked control system



$\hat{\mathbf{y}}(t)$: the sequence of sensor values.

Time diagram

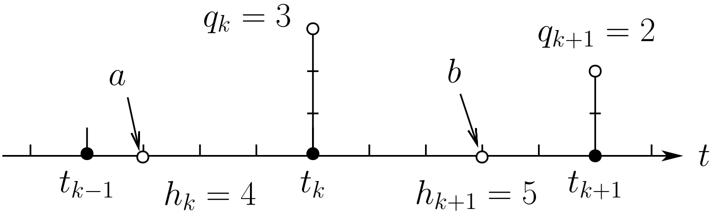
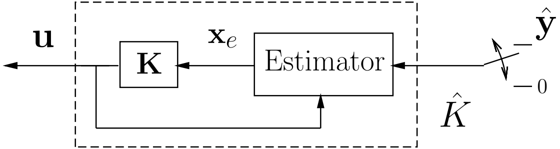


Figure: Time diagram of NCS with buffer status at t_k and t_{k+1}, assuming $q_{\max} = 3$.

t_k: Update instant.
h_k: Stochastic update intervals.
$\hat{\mathbf{y}}(t_k) := \{\mathbf{y}(t_k), \mathbf{y}(t_k - 1), \cdots, \mathbf{y}(t_k - q_k + 1)\}$

Controller Mechanism



Switched estimate scheme:

- Open loop ($t \neq t_k$):
  $$\mathbf{x}_e(t+1) = \hat{\mathbf{A}}\mathbf{x}_e(t) + \hat{\mathbf{B}}\mathbf{u}(t).$$
- Closed loop ($t = t_k$):
  $$\mathbf{x}_e(t_k+1) = \begin{cases} \hat{\mathbf{y}}(t_k) \\ \mathbf{x}_e(t_k - q_k + 1) \\ \{\mathbf{u}(t_k), \mathbf{u}(t_k-1), \cdots, \mathbf{u}(t_k - q_k + 1)\} \end{cases}$$

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Proposed NCS

The dynamic of NCS:

$$\mathbf{z}(t) = \Lambda_0^{t-t_k-1}\Big(\prod_{j=1}^{k}\mathbf{M}(h_j, q_j)\Big)\mathbf{z}_0, \quad t \in (t_k, t_{k+1}]$$

$\mathbf{M}(h_j, q_j)$: State transition matrix for each update interval.

Stability Analysis

Probability distribution for h_k:

- No particular:
    - Arbitrary transmission
    - Lyapunov asymptotically stability
- Markovian:
    - Markovian transmission
    - Mean square stability

Numerical Example

3^{rd}-order unstable plant with Markovian transmission

- given $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, $\hat{\mathbf{C}}$, $\mathbf{K}$, $\mathbf{L}$.
- maximal update interval $N = 8$.
- transition probability matrix $\Gamma \in \mathbb{R}^{8 \times 8}$.
- maximal buffer capacity $q_{max} = 5$.

Simulation Result

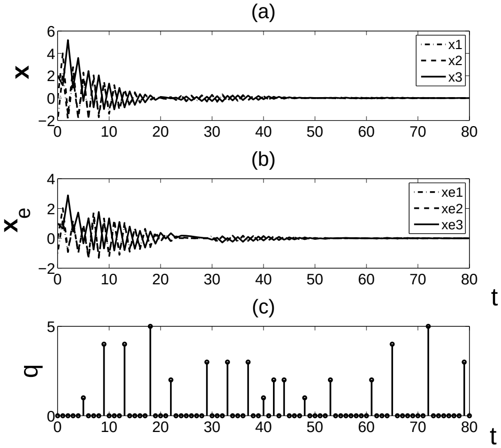

Figure: (a) state trajectories of the plant $\hat{P}$, (b) state trajectories of the estimator, and (c) sequences of the update instants $\{t_k\}$ and the buffer lengths $\{q_k\}$.

Confirmation of Mean Square Stability

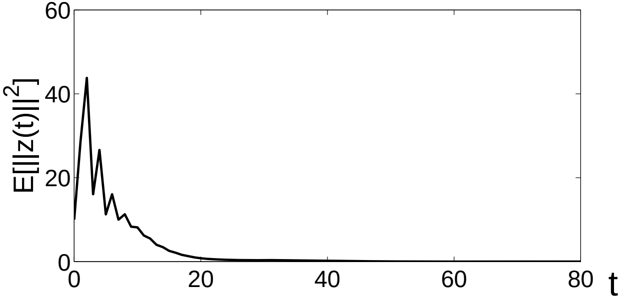

Figure: $\lim_{t\to\infty} E[\|\mathbf{z}(t,\mathbf{z}_0)\|^2] = 0$, where $E[\|\mathbf{z}(t,\mathbf{z}_0)\|^2]$ was calculated by averaging over 200 simulation runs.

Summary

- Stability properties for arbitrary/Markovian transmission.
- Stochastic update intervals.
- Random buffer capacity.

Thank You!