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A Reduced-Complexity Demapping Algorithm for BICM-ID Systems

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***Abstract*—As a highly efficient decoding and demodulation scheme, bit-interleaved coded modulation (BICM) is widely adopted in modern communication systems. In order to enhance the attainable spectral efficiency, usually, high-order modulation schemes are used for BICM systems. When BICM is combined with iterative decoding (BICM-ID), it is capable of further improving the achievable receiver performance. However, the complexity of the standard max-sum approximation of the maximum *a posteriori* probability in log-domain (Max-Log-MAP) invoked by the iterative demapper is on the order of 2^m or $\mathsf{O}(2^m)$ for a 2^m-ary modulation constellation having m bits per symbol, which may become excessive for high-order BICM-ID systems. The existing simplified algorithms employed for noniterative demappers are based on exploiting the constellation's symmetry, which is no longer retained upon the introduction of the *a priori* information in BICM-ID systems. Hence, in this paper, a simplified iterative demapping algorithm is proposed to substantially reduce the demapping complexity for a binary-reflected Gray-labeled constellation. Our detailed analysis shows that the simplified demapping scheme proposed for BICM-ID reduces the computational complexity to $\mathsf{O}(m)$. We demonstrate that this dramatic computational complexity only imposes modest performance degradation with respect to that of the optimal high-complexity Max-Log-MAP scheme.**

***Index Terms*—Bit-interleaved coded modulation with iterative decoding (BICM-ID), iterative demapper, maximum *a posteriori* probability in log-domain demapping, pulse amplitude modulation (PAM), quadrature amplitude modulation (QAM).**

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I. Introduction

The original concept of bit-interleaved coded modulation (BICM) was presented by Zehavi [1], who found that the performance of coded modulation over Rayleigh fading channels can be considerably improved by adopting bit interleaving. Based on his work, a set of theoretical modeling and analysis tools has been developed for BICM for other applications as well [2]. In 1998, Li and Ritcey [3] improved BICM with the aid of iterative decoding (BICM-ID). In the same year, ten Brink *et al.* [4] provided further insights on enhancing the performance of BICM-ID at reduced complexity. Therefore, state-of-the-art communication systems often employ BICM to improve the attainable performance, as exemplified by the second-generation digital terrestrial television broadcasting standard (DVB-T2) system [5]. For the sake of achieving high bandwidth efficiency, typically high-order quadrature amplitude modulation (QAM) [6] is recommended for both noniterative BICM and BICM-ID systems. The Gray-labeled mapping is also widely applied to high-order modulation constellation schemes to reduce the bit errors since two adjacent Gray-labeled constellation points differ in only one bit [7]. The DVB-T2 standard [5], for example, supports binary-reflected Gray-labeled modulation. However, high-order BICM and BICM-ID systems impose high-complexity when adopting the optimal log-domain maximum *a posteriori* probability (Log-MAP) demapping algorithm [8]. Although the max-sum approximation of the Log-MAP (Max-Log-MAP) algorithm [9] is capable of substantially reducing the associated computational complexity, it is still on the order of $\mathsf{O}(2^m)$, where 2^m is the size of the modulation constellation employed that contains m bits per symbol.

Several simplifications of the Max-Log-MAP algorithm have been proposed for high-order constellations. Tosato and Bisaglia [10] found that 2^m-QAM symbols relying on Gray labeling can be divided into two independent $2^{m/2}$-ary pulse amplitude modulation (PAM) constellations, and with the aid of this partitioning, the complexity of the noniterative demapper can be reduced to $\mathsf{O}(2^{m/2})$ without any performance degradation. In [10], the complexity of the simplified QAM demapper is further reduced to $\mathsf{O}(m)$ by adopting a piecewise linear approximation, but this simplification results in modest performance degradation. Since the *a priori* information necessary for iterative demapping is not included in the simplified algorithms in [10], it cannot be readily applied to BICM-ID systems. The simplified demapping algorithm in [11] was conceived for universal Gray-labeled constellations, which reduces the complexity to $\mathsf{O}(m)$ by exploiting the symmetry of Gray-labeled constellations while attaining exactly the same performance as the Max-Log-MAP algorithm for QAM, PAM, and phase-shift keying (PSK) constellations. However, this algorithm cannot be applied to the iterative demapper since the symmetry of Gray-labeled constellations is no longer satisfied after taking into account the input *a priori* information. Further efforts invested in reducing the complexity of BICM-ID systems were reported in [12], where hard-feedback-based iterative decoding was used to reduce the complexity of BICM-ID combined with frequency-shift keying modulation, whereas in [13], a mapping scheme combined with low-density parity-check (LDPC) coding is proposed to merge the demapping operation into the process of LDPC decoding. However, the methods in [12] and [13] are only applicable to orthogonal modulation schemes. By only allowing the extrinsic information for systematic bits to be fed back to the demodulator, the scheme proposed in [14] was shown to reduce the complexity of BICM-ID schemes, but the objective of this scheme is to reduce the decoding complexity of turbo coding rather than to simplify the demapping algorithm conceived for universal modulation and coding schemes.

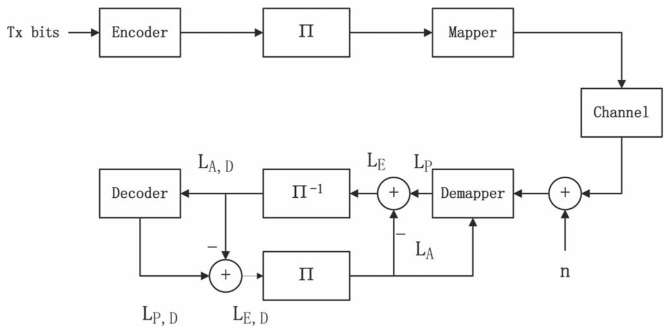

Fig. 1. Standard BICM-ID system structure.

Against the above background, in this paper, a simplified iterative soft demapper is conceived for Gray-labeled modulation, which dramatically reduces the computational complexity of BICM-ID systems. Specifically, we introduce a low-complexity technique of transforming the *a posteriori* decoding information into a modulated symbol to form an equivalent received symbol, which enables us to exploit the symmetry of the original Gray-labeled constellation for simplifying the maximization of the log-likelihood values required for iterative demapping. We prove that, for a 2^m-ary constellation, the complexity of the proposed iterative demapper is only on the order of $\mathrm{O}(m)$, as opposed to the $\mathrm{O}(2^m)$ complexity of the standard Max-Log-MAP algorithm. This substantial complexity reduction is achieved at a modest performance loss for both additive white Gaussian noise (AWGN) and Rayleigh fading channels, as it is clearly demonstrated in our extensive simulations.

The rest of this paper is organized as follows. Section II briefly reviews the typical BICM-ID system model and the standard iterative Max-Log-MAP demapping algorithm. In Section III, our new simplified iterative soft demapping algorithm is detailed and its computational complexity is analyzed. In Section IV, based on an extensive simulation study, the performances of both the proposed low-complexity iterative demapper and the conventional Max-Log-MAP iterative demapper are quantified and compared for Gray-labeled QAM for transmission over both AWGN and Rayleigh fading channels. Our conclusions are drawn in Section V.

The following notations are adopted throughout our discussions. Sets are represented by uppercase calligraphic letters, e.g., $\mathcal{X}$, whereas column vectors are represented by boldface lowercase letters, e.g., $\mathbf{b}$ with the ith element of $\mathbf{b}$ being denoted as b_i. The exception is the log-likelihood ratio (LLR) vector, which is denoted by $\mathbf{L}$. Uppercase letters denote random variables (RVs), e.g., X, and the corresponding lowercase letters represent their realizations, e.g., x. $P(x)$ is used for the probability mass function (PMF) of a discrete RV X, whereas $p(x)$ denotes the probability density function (PDF) of a continuous RV X. Additionally, $P(y|x)$ represents the conditional PMF of $Y = y$ given $X = x$, whereas $p(y|x)$ represents the conditional PDF of $Y = y$ given $X = x$. $E(y|x)$ represents the conditional expectation of $Y = y$ given $X = x$. The magnitude operator is denoted by $|\cdot|$, whereas the transpose operator is represented by $(\cdot)^T$. Additionally, $\oplus$ represents the bitwise exclusive-OR operation, whereas the imaginary axis is defined by $\mathrm{j} = \sqrt{-1}$.

II. Bit Interleaved Coded Modulation With Iterative Decoding System Model With Max-Log-Maximum *a posteriori* Demapper

The system model of BICM-ID [4] is shown in Fig. 1. At the transmitter, the bits are interleaved after channel encoding. After grouping the interleaved bits into m-length bit vectors, each bit vector $\mathbf{b} = [b_0 \quad b_1 \quad \cdots b_{m-1}]^T$ is mapped onto a complex-valued symbol $x \in \mathcal{X}$, where $\mathcal{X} = \{x_k, 0 \le k < 2^m - 1\}$ denotes the constellation set of the specific 2^m-order modulation scheme. The channel is modeled as $y = hx + n$, where y is the received symbol, h denotes the complex-valued channel impulse response (CIR), and n denotes the complex-valued channel AWGN with zero mean and a variance of $\sigma^2/2$ per dimension.

At the receiver, the iterative soft demapper calculates the *a posteriori* information $\mathbf{L}_P = [L_{P_0} \quad L_{P_1} \quad \cdots \quad L_{P_{m-1}}]^T$, given y and the input *a priori* information $\mathbf{L}_A = [L_{A_0} \quad L_{A_1} \quad \cdots \quad L_{A_{m-1}}]^T$, where the ith element of $\mathbf{L}_P$, which is denoted as L_{P_i}, is defined as the LLR of

$$L_{P_i} = \log \frac{P\big(b_i = 0|y, \mathbf{L}_A\big)}{P\big(b_i = 1|y, \mathbf{L}_A\big)}. \tag{1}$$

The demapping extrinsic information $\mathbf{L}_E = \mathbf{L}_P - \mathbf{L}_A$ is then deinterleaved and passed to the soft decoder as its input *a priori* information $\mathbf{L}_{A,D}$. After soft decoding, the decoder extrinsic information $\mathbf{L}_{E,D}$, defined as the difference between the decoder's *a posteriori* information $\mathbf{L}_{P,D}$ and the decoder's *a priori* information $\mathbf{L}_{A,D}$, is transferred to the demapper through the interleaver as its *a priori* feedback information $\mathbf{L}_A$ for the next iteration.

According to the standard Max-Log-MAP iterative demapping algorithm [15], the *a posteriori* LLR L_{P_i}, which is defined in (1), can be approximated as

$$\begin{aligned} L_{P_i} &= \log \frac{\sum\limits_{x \in \mathcal{X}_i^{(0)}} P(y|x)P(x|\mathbf{L}_A)}{\sum\limits_{x \in \mathcal{X}_i^{(1)}} P(y|x)P(x|\mathbf{L}_A)} \\ &\approx \log \frac{\max\limits_{x \in \mathcal{X}_i^{(0)}} \{P(y|x)P(x|\mathbf{L}_A)\}}{\max\limits_{x \in \mathcal{X}_i^{(1)}} \{P(y|x)P(x|\mathbf{L}_A)\}} \end{aligned} \tag{2}$$

for $i = 0, 1, \ldots, m-1$, where $\mathcal{X}_i^{(b)}$ represents the constellation subset of $\mathcal{X}$ with the ith labeled bit $b_i = b \in \{0, 1\}$.

Denoting the labeled bit vector of each constellation point x by $\mathbf{b}_x = [b_{x_0} \quad b_{x_1} \quad \cdots \quad b_{x_{m-1}}]^T$, the *a posteriori* probability $P(x|\mathbf{L}_A)$ is formulated as [16]

$$\begin{aligned} P(x|\mathbf{L}_A) &= \prod_{i=0}^{m-1} P\big(b_i = b_{x_i}\big) \\ &= \prod_{i=0}^{m-1} \frac{P\big(b_i = b_{x_i}\big)}{\sqrt{P(b_i = 0)P(b_i = 1)}} \\ &\quad \times \prod_{i=0}^{m-1} \sqrt{P(b_i = 0)P(b_i = 1)} \\ &= \exp\left(\frac{1}{2}\sum_{i=0}^{m-1} s_{x_i} \log \frac{P(b_i = 0)}{P(b_i = 1)}\right) \\ &\quad \times \prod_{i=0}^{m-1} \sqrt{P(b_i = 0)P(b_i = 1)} \\ &= \exp\left(\frac{1}{2}\mathbf{s}_x^T \mathbf{L}_A\right) \prod_{i=0}^{m-1} \sqrt{P(b_i = 0)P(b_i = 1)} \end{aligned} \tag{3}$$

where $\mathbf{s}_x = [s_{x_0} \quad s_{x_1} \quad \cdots \quad s_{x_{m-1}}]^T = \mathbf{1} - 2\mathbf{b}_x$, and $\mathbf{1}$ denotes the m-length vector, with all the elements being 1.

Since the channel noise n obeys a circular complex Gaussian distribution, the conditional PDF $p(y|x)$ is expressed as

$$p(y|x) = \frac{1}{\pi\sigma^2}\exp\left(-\frac{|y-hx|^2}{\sigma^2}\right). \quad (4)$$

By combining (2) with (4), the ith *a posteriori* information L_{P_i} is simplified to [16]

$$L_{P_i} = \max_{x\in\mathcal{X}_i^{(0)}}\left\{-\frac{1}{\sigma^2}|y-hx|^2 + \frac{1}{2}\mathbf{s}_x^T\mathbf{L}_A\right\} - \max_{x\in\mathcal{X}_i^{(1)}}\left\{-\frac{1}{\sigma^2}|y-hx|^2 + \frac{1}{2}\mathbf{s}_x^T\mathbf{L}_A\right\}. \quad (5)$$

It is shown in (5) that the maximization operations of the Max-Log-MAP algorithm require the calculation of the squared Euclidean distances $|y-hx|^2$ for all the 2^m constellation points $x\in\mathcal{X}$, whereas the calculation of $\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A$ only involves changing signs and additions. Additionally, maximizing $-(1/\sigma^2)|y-hx|^2 + (1/2)\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A$ for $x\in\mathcal{X}_i^{(b)}$ with $b\in\{0,1\}$ requires 2^m-2 paired comparisons. Consequently, the computational complexity of the standard Max-Log-MAP iterative demapping algorithm is of $\mathsf{O}(2^m)$, which becomes excessive for high-order modulation constellations. This severely limits the application of BICM-ID in systems employing high-order modulation schemes.

It is also worth emphasizing that by setting $\mathbf{s}_x^T\mathbf{L}_A = 0$ in (5), it is reduced to the LLR calculation of the Max-Log-MAP algorithm for noniterative BICM systems since there is no iteration between the soft decoder and the soft demapper in noniterative systems. Therefore, the universal simplified algorithm for noniterative systems developed in [11] cannot be applied to simplify the calculation of (5).

III. Proposed Simplified Demapper for Bit Interleaved Coded Modulation With Iterative Decoding System

Obviously, either $\max_{x\in\mathcal{X}_i^{(0)}}\{-(1/\sigma^2)|y-hx|^2+(1/2)\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A\}$ or $\max_{x\in\mathcal{X}_i^{(1)}}\{-(1/\sigma^2)|y-hx|^2+(1/2)\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A\}$ is equal to $\max_{x\in\mathcal{X}}\{-(1/\sigma^2)|y-hx|^2+(1/2)\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A\}$. Let $x^* = \arg\max_{x\in\mathcal{X}}\{-(1/\sigma^2)|y-hx|^2 + (1/2)\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A\}$ with its associated bit vector denoted by $\mathbf{b}^* = [b_1^* \quad b_2^* \quad \cdots \quad b_{m-1}^*]^T$. Then the LLR (5) must contain the term $-(1/\sigma^2)|y-hx^*|^2+(1/2)\mathbf{s}_{x^*}^{\mathrm{T}}\mathbf{L}_A$, whereas its other term is given by $\max_{x\in\mathcal{X}_i^{(\overline{b_i^*})}}\{-(1/\sigma^2)|y-hx|^2 + (1/2)\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A\}$, where $\overline{b_i^*} = 1-b_i^*$. If we introduce the notation $x^*_{i,\overline{b_i^*}} = \arg\max_{x\in\mathcal{X}_i^{(\overline{b_i^*})}}\{-(1/\sigma^2)|y-hx|^2 + (1/2)\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A\}$, the LLR (5) can be expressed as

$$L_{P_i} = \frac{1-2b_i^*}{\sigma^2}\left(-|y-hx^*|^2 + \frac{\sigma^2}{2}\mathbf{s}_{x^*}^{\mathrm{T}}\mathbf{L}_A + \left|y-hx^*_{i,\overline{b_i^*}}\right|^2 - \frac{\sigma^2}{2}\mathbf{s}^{\mathrm{T}}_{x^*_{i,\overline{b_i^*}}}\mathbf{L}_A\right). \quad (6)$$

A. Proposed Simplification

For the Max-Log-MAP iterative demapping algorithm, the constellation points x^* and $x^*_{i,\overline{b_i^*}}$ are determined by y and $\mathbf{L}_A$. To reduce the associated complexity, our proposed simplified algorithm merges the *a priori* information y and $\mathbf{L}_A$ into an equivalent received symbol $\widehat{y}$ and determines x^* and $x^*_{i,\overline{b_i^*}}$ approximately by maximizing $-|\widehat{y}-hx|^2$, instead of maximizing $-|y-hx|^2 + (\sigma^2/2)\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A$. To maintain the accuracy of the approximation, the conditional expectation $E(x|\mathbf{L}_{P,D})$ is selected as the estimated transmitted symbol $\widehat{x}$ because the LLR information of the decoder $\mathbf{L}_{P,D}$ in the previous iteration includes the *a priori* information of y and $\mathbf{L}_A$. Additionally, a low-complexity method is adopted for calculating the conditional expectation $E(x|\mathbf{L}_{P,D})$.

More specifically, to acquire the value of $E(x|\mathbf{L}_{P,D})$, the conditional probabilities of b_i and x_k are first determined according to the *a priori* LLR information $\mathbf{L}_{P,D}$. In particular, the conditional probability of the ith bit b_i is calculated as [16]

$$P(b_i=0|\mathbf{L}_{P,D}) = \frac{\exp(L_{P,D_i})}{1+\exp(L_{P,D_i})} \quad (7)$$

$$P(b_i=1|\mathbf{L}_{P,D}) = \frac{1}{1+\exp(L_{P,D_i})}. \quad (8)$$

By denoting $\mathbf{b}_k = [b_{k_0} \quad b_{k_1} \quad \cdots \quad b_{k_{m-1}}]^T$ as the bit vector that maps onto the corresponding constellation point x_k and assuming that the elements of $\mathbf{b}_k$ are uncorrelated, we have the conditional symbol probability $P(x=x_k|\mathbf{L}_{P,D})$ given by

$$\log P(x=x_k|\mathbf{L}_{P,D}) = \sum_{i=0}^{m-1}\log P(b_i=b_{k_i}|\mathbf{L}_{P,D}). \quad (9)$$

The equivalent transmitted symbol $\widehat{x}$ is defined as the following conditional expectation:

$$\widehat{x} = E(x|\mathbf{L}_{P,D}) = \sum_{k=0}^{2^m-1} x_k P(x=x_k|\mathbf{L}_{P,D}). \quad (10)$$

By taking into account the effect of the CIR h, the equivalent received symbol $\widehat{y}$ is estimated as

$$\widehat{y} = E(hx+n|\mathbf{L}_{P,D}) = h\widehat{x}. \quad (11)$$

Finding x^* and $x^*_{i,\overline{b_i^*}}$ in (6) can be approximated by finding the elements in both $\mathcal{X}$ and $\mathcal{X}_i^{(\overline{b_i^*})}$ that maximize the term $-|\widehat{y},-hx|^2 = -|h|^2|\widehat{x}-x|^2$. This maximization is equivalent to determining the nearest constellation points of the set $\mathcal{X}$ and the subset $\mathcal{X}_i^{(\overline{b_i^*})}$ to the estimated symbol point $\widehat{x}$, respectively, in terms of the squared Euclidean distance metric.

Let us now consider the binary-reflected Gray-labeled constellation point x_k and denote the binary vector representation of the symbol index k with the least significant bit as the last element by $\mathbf{c}_k = [c_0^k \quad c_1^k \quad \cdots \quad c_{m-1}^k]^T$. Then the bit vector $\mathbf{b}_k$ can be defined as [11]

$$\mathbf{b}_k = [c_0^k \quad c_1^k \quad \cdots \quad c_{m-1}^k]^T \oplus [0 \quad c_0^k \quad \cdots \quad c_{m-2}^k]^T. \quad (12)$$

Assume that the indices k^* and k_i^* satisfy the equations $x^* = x_{k^*}$ and $x^*_{i,\overline{b_i^*}} = x_{k_i^*}$, respectively. For any specific constellation, k_i^* is uniquely determined by k^* and i [11] so that it is no longer necessary to calculate the squared Euclidean distances from $\widehat{x}$ with respect to all the elements in the subset $\mathcal{X}_i^{(\overline{b_i^*})}$. Let us now describe how our proposed simplified algorithm finds x^* and $x^*_{i,\overline{b_i^*}}$, i.e., determining k^* and k_i^*, for any 2^m-ary Gray-labeled constellation.

1) PAM Demapper: Since PAM symbols are real valued, a binary search can be used to find x^*. Without loss of generality, the symbols in the PAM constellation set $\mathcal{X}$ are sorted in ascending order. The threshold $\delta_1 = (x_{2^{m-1}-1} + x_{2^{m-1}}/2)$ divides $\mathcal{X}$ into the two symmetrical subsets $\mathcal{X}_{\text{Left}} = \{x_k,\ 0 \le k < 2^{m-1}\}$ and $\mathcal{X}_{\text{Right}} = \{x_k,\ 2^{m-1} \le k < 2^m\}$. If $\widehat{x} < \delta_1$, then we have $x^* \in \mathcal{X}_{\text{Left}}$; otherwise, we have $x^* \in \mathcal{X}_{\text{Right}}$. Upon repeating the previous division and comparison on the subset that contains x^*, we arrive at $x^* = x_{k^*}$ at the mth repetition of the procedure because each of the corresponding subsets $\mathcal{X}_{\text{Left}}$ and $\mathcal{X}_{\text{Right}}$ contains only a single point. For a Gray-labeled PAM constellation, k_i^* is calculated according to [11]

$$k_i^* = 2^{m-i-1} - c_i^{k^*} + \sum_{j=0}^{i-1} c_j^{k^*} 2^{m-j-1}. \quad (13)$$

2) QAM Demapper: The procedure of determining x_{k^*} and $x_{k_i^*}$ is the same as that for the Gray-labeled PAM demapper previously described since the 2^m-ary QAM symbol set can be viewed as consisting of the two independent (in-phase and quadrature phase) $2^{m/2}$-ary PAMs. Given the estimated $\widehat{x} = \widehat{x}_I + \mathrm{j}\widehat{x}_Q$, where both $\widehat{x}_I$ and $\widehat{x}_Q$ are real valued $2^{m/2}$-ary PAM symbols, we can calculate the corresponding PAM symbol indices k_I^* and $k_{i,I}^*$, as well as k_Q^* and $k_{i,Q}^*$, and consequently obtain

$$x^* = x_{k_I^*} + \mathrm{j}x_{k_Q^*} \quad (14)$$

$$x^*_{i,\overline{b_i^*}} = x_{k_{i,I}^*} + \mathrm{j}x_{k_{i,Q}^*}. \quad (15)$$

3) PSK Demapper: The 2^m-ary PSK constellation set can be expressed as $\mathcal{X} = \left\{x_k = \sqrt{E_s}\exp\left(\mathrm{j}(2k+1)\pi/2^m\right) 0 \le k < 2^m\right\}$, where E_s denotes the signal power. Let $\widehat{y} = \rho_{\widehat{y}} \exp\left(\mathrm{j}\varphi_{\widehat{y}}\right)$, where $\rho_{\widehat{y}}$ and $\varphi_{\widehat{y}}$ are the amplitude and phase of $\widehat{y}$, respectively. Similar to the noniterative simplified demapper for the Gray-labeled PSK [11], the index k^* of the constellation point x^*, which has the nearest phase to $\varphi_{\widehat{y}}$, can be found by a binary search with the aid of m comparisons. Furthermore, k_i^* is calculated according to [11]

$$k_i^* = \begin{cases} \overline{c_0^{k^*}} 2^{m-1} + \overline{c_1^{k^*}}\left(2^{m-1} - 1\right), i = 0 \\ 2^{m-i-1} - c_i^{k^*} + \sum_{j=0}^{i-1} c_j^{k^*} 2^{m-j-1}, i > 0. \end{cases} \quad (16)$$

4) APSK Demapper: The Gray-labeled amplitude and PSK (APSK) modulation scheme can be viewed as the product of a PAM scheme and a PSK scheme [11]. By applying the PAM and PSK demappers, we can find the indices k_{PAM}^* and $k_{i,\text{PAM}}^*$, as well as k_{PSK}^* and $k_{i,\text{PSK}}^*$, for the two corresponding PAM and PSK decomposition signals, respectively, and consequently find the indices k^* and k_i^* for the APSK symbols x_{k^*} and $x_{k_i^*}$.

By substituting $x^* = x_{k^*}$ and $x^*_{i,\overline{b_i^*}} = x_{k_i^*}$ in (6), we have arrived at the approximate ith bit demapping information L_{P_i}. Note that, in this approximation, x^* and $x^*_{i,\overline{b_i^*}}$ are selected according to the estimated symbol $\widehat{x}$, instead of the received signal y and the *a priori* information $\mathbf{L}_A$. Therefore, the term $-|y - hx|^2 + (1/2)\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A$ for $x = x_{k^*}$ or $x = x_{k_i^*}$ may be smaller than the real maximum of $-|y - hx|^2 + (1/2)\mathbf{s}_x^{\mathrm{T}}\mathbf{L}_A$ over the corresponding subset $\mathcal{X}_i^{(0)}$ or $\mathcal{X}_i^{(1)}$, respectively. Considering that the estimated symbol $\widehat{y}$ is nearer hx_{k^*} and $hx_{k_i^*}$ than y, we use a weighted average as subsequently given to substitute for y in (6)

$$y^* = \alpha\widehat{y} + (1 - \alpha)y \quad (17)$$

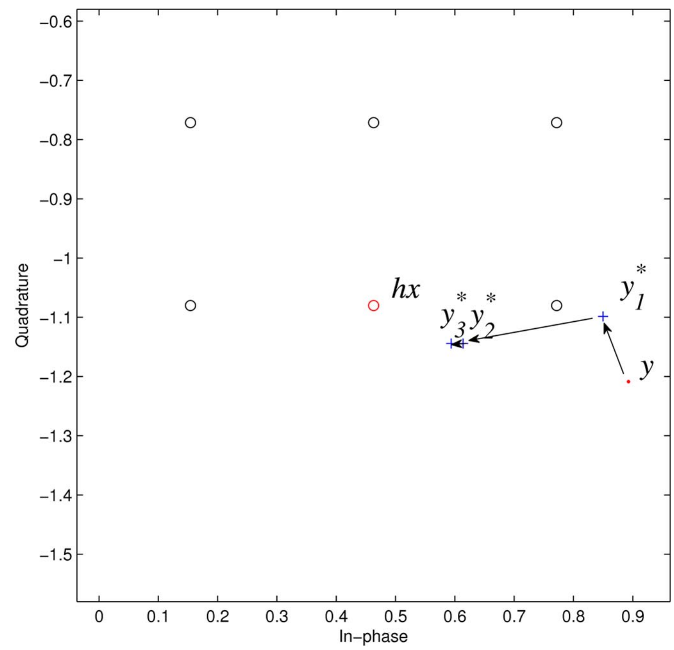

Fig. 2. Illustrative movement of the compensated symbol y^* with $\alpha = 0.5$ for a 64QAM AWGN channel.

where $0 \le \alpha \le 1$ denotes the compensating coefficient. In particular, $\alpha = 0$ represents no substitution, whereas $\alpha = 1$ corresponds to using $\widehat{y}$ to substitute for y in (6).

The example given in Fig. 2 shows the beneficial effect of (17), where the arrow indicates the movement of the compensated symbol y^* with $\alpha = 0.5$ after each iterative demapping and decoding operation, whereas the symbol y_l^* denotes the value of y^* after the lth iterative decoding step. The received symbol y is far away from the transmitted symbol hx since the channel signal-to-noise ratio[1] (SNR) is set to as low as 10.6 dB in this example. After three demapping and decoding iterations, y^* moves to a position much closer to the transmitted symbol hx. When the error occurs in the approximately estimated x^* and $x^*_{i,\overline{b_i^*}}$, the estimated LLR L_{P_i} based on them will be smaller than the true optimal LLR value. Since y^* is closer to the transmitted symbol hx than y, substituting y^* for y in (6) will amplify the estimated LLR value, and this effectively compensates the error in estimating x^* and $x^*_{i,\overline{b_i^*}}$ by the proposed simplified method.

Appropriate value of the coefficient α depends on the specific modulation and coding scheme employed and on the channel condition. We present a strategy for setting the value of α based on extrinsic information transfer (EXIT) chart analysis [17]. Fig. 3 is an illustrative example of selecting the value of α based on EXIT chart analysis for the case of 64QAM and LDPC coding for transmission over the AWGN channel, given the channel SNR of 10 dB. In Fig. 3, IA1/IA2 represent the input extrinsic information entered into the demapper/decoder, whereas IE1/IE2 denote the extrinsic information output from the demapper/decoder. The EXIT curves of the proposed simplified iterative demapper associated with various values of α, together with the inverted EXIT curve of the LDPC decoder based on the min-sum decoding algorithm [18], are plotted in Fig. 3. It is shown in Fig. 3 that for this case, the accuracy of the LLR value calculation based on the estimated x^* and $x^*_{i,\overline{b_i^*}}$ without compensation ($\alpha = 0$) is

[1]The SNR is defined as the ratio of the average symbol energy to the noise power spectral density, namely, $\text{SNR} = (E(x^2)/\sigma^2)$.

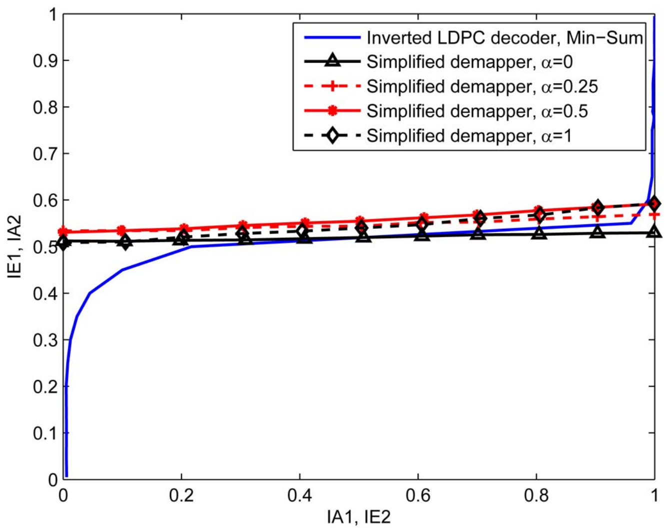

Fig. 3. Determining the value of α based on EXIT chart analysis for the case of 64QAM and LDPC coding scheme over the AWGN channel with a channel SNR of 10 dB.

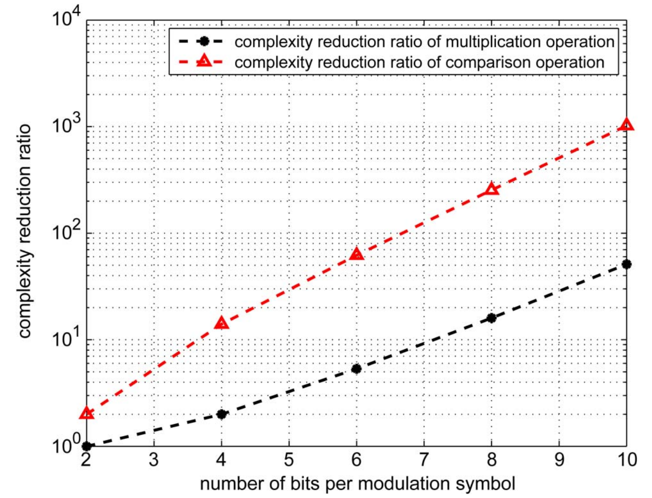

Fig. 4. Complexity reduction ratio of the proposed simplified demapping algorithm compared with the Max-Log-MAP algorithm as a function of m.

insufficient since the iterative demapping and decoding will be stalled at the point of the input *a priori* information of 0.5. For all the other values of α, the iterative demapping and decoding can converge to the point of the input *a priori* information in the vicinity of 1.0. Since the demapper EXIT curve associated with $\alpha = 0.5$ is above the other curves, setting $\alpha = 0.5$ enables the proposed simplified iterative demapper to achieve the optimal convergence performance, i.e., to best compensate for the error imposed on approximating the LLR value in this case.

In theory, the optimal α selection needs to be repeated for different constellation sizes, code rates, and/or channel types. Our experience suggests that the channel type, e.g., AWGN or Rayleigh fading, has most significant influence on the optimal α value. By contrast, for the same channel type, the optimal α values differ very little for different constellation sizes and/or code rates. Since in practice we only need to choose an appropriate, rather than an optimal, α value, the α selection procedure does not have to be repeated for all the constellation sizes and code rates.

B. Summary of the Proposed Algorithm and Complexity Analysis

Our proposed simplified iterative demapping algorithm consists of the following three steps.

1) Estimate the equivalent transmitted symbol $\widehat{x}$ and received symbol $\widehat{y}$, according to (7)–(11).
2) Determine the estimated x^* and $x^*_{i,\overline{b^*_i}}$ based on $\widehat{x}$, as detailed in the previous section.
3) Compensate the error in the estimated x^* and $x^*_{i,\overline{b^*_i}}$ by applying (17), and then calculate the LLR L_{P_i} by substituting the compensated symbol y^* for y in (6).

In the first decoding iteration, the *a priori* information is zero. Therefore, the calculation of *a posteriori* information L_{P_i} based on (5) is exactly the same as in the Max-Log-MAP noniterative demapper. Thus, we adopt the noniterative simplified demapping algorithm [11] to calculate the *a posteriori* information L_{P_i} at the first iteration. From the second iteration onward, the proposed algorithm described in the three steps is used to fulfill the iterative demapping.

In step 1, the estimated symbol $\widehat{x}$ is calculated according to (7)–(10). The logarithmic function evaluation of $\log P\left(b_i = b_{k_i}|\mathbf{L}_{P,D}\right)$ can be implemented with the aid of two lookup tables, which adopt L_{P,D_i} as the address bits and store the corresponding values of $\log(\exp\left(L_{P,D_i}\right)/1 + \exp\left(L_{P,D_i}\right))$ and $\log(1/1 + \exp\left(L_{P,D_i}\right))$, respectively. A third lookup table is needed to implement the function evaluation of $P\left(b_i = b_{k,i}|\mathbf{L}_{P,D}\right) = \exp(\log P\left(b_i = b_{k,i}|\mathbf{L}_{P,D}\right))$. In (10) since the constellation point x is constant, we can use the shifting operation and addition to achieve the multiplication of variable $P(x = x_k|\mathbf{L}_{P,D})$ and constant x. In step 2, x^* and $x^*_{i,\overline{b^*_i}}$ are found. Similar to the complexity analysis in [11], for the case of 2^m-ary Gray-labeled QAM, for example, the determination of k^*_I and k^*_Q can be achieved by a binary search algorithm with the aid of as few as m comparison operations, whereas determining $k^*_{i,I}$ and $k^*_{i,Q}$ only requires the binary representations of k^*_I and k^*_Q, respectively, and a few addition operations. The computational requirements for other modulation schemes are similar, and they can be analyzed according to [11]. In step 3, implementing the compensation operation (17) and calculating the LLR value (6) require $4m$ multiplications. Thus, our proposed iterative demapper requires $4m$ multiplications, m comparison operations, and a few additions, as well as the cost of a few memory resources. Clearly, it has computational complexity on the order of $\mathsf{O}(m)$. To provide insight into the dramatic complexity reduction of our simplified algorithm, in comparison with the Max-log-MAP algorithm, Fig. 4 shows the complexity reduction ratio curves both in terms of the multiplication and comparison operations, with m varied from 2 to 10.

IV. Simulation Results

Our proposed simplified algorithm applies an approximate method for calculating the iterative demapping soft information, and consequently, it inevitably suffers from a performance loss, in comparison with the standard high-complexity optimal Max-Log-MAP iterative demapper. A simulation study was carried out to quantitatively evaluate this performance gap between the proposed simplified iterative algorithm and the standard Max-Log-MAP iterative demapper. The parameters of the simulated BICM-ID system were as follows.

- Constellation: Gray-labeled 64QAM and 256QAM;

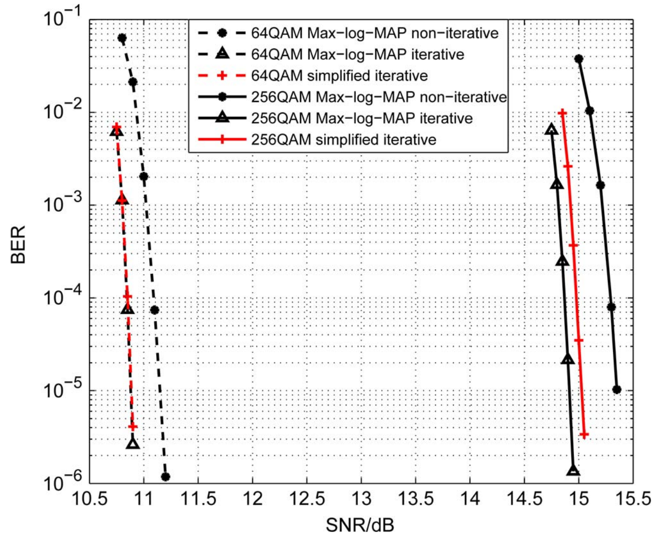

Fig. 5. BER performance comparison over the AWGN channel.

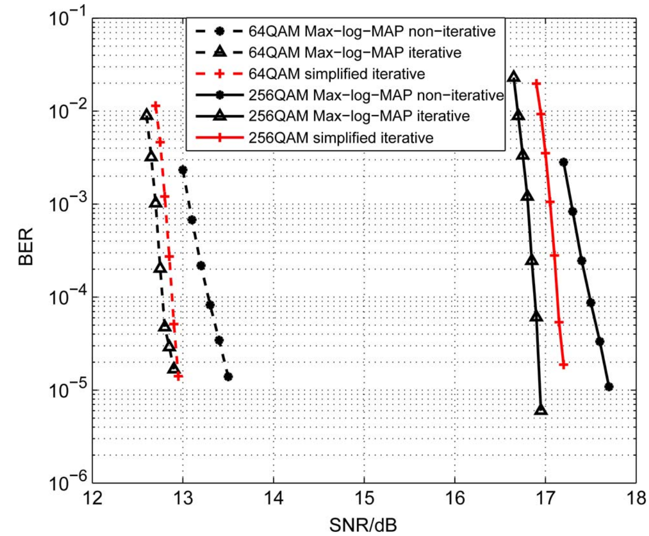

Fig. 6. BER performance comparison over the Rayleigh fading channel.

- Channel: AWGN and independently identically distributed Rayleigh fading channels;
- Coder and decoder: The 1/2-rate 64 800-bit LDPC code from DVB-T2 [5] and the normalized min-sum decoding algorithm [18] with the maximum decoding iterations of 25 and the normalization factor of 1/0.875;
- Interleaver: No interleaver used in either the transmitter or the receiver since the row and column of LDPC code check matrix were interleaved;
- Demapper: The Max-Log-MAP noniterative demapper, the Max-Log-MAP iterative demapper, and the proposed simplified iterative demapper. The number of iterations was set to 4 for all the three algorithms. The compensating coefficient α for the proposed algorithm was set to 1/2 and 3/16 for the AWGN and Rayleigh fading channels, respectively.

The bit error rate (BER) performances for transmission over the AWGN channel are shown in Fig. 5. For the 64QAM scheme, the proposed simplified iterative demapper achieves almost the same performance as the standard Max-Log-MAP iterative demapper, whereas it enjoys an advantage of 0.25-dB SNR gain over the noniterative Max-Log-MAP demapper at the BER level of 10^{-5}. As for the 256QAM scheme, the simplified iterative demapper suffers from a performance loss of less than 0.15 dB at the BER level of 10^{-5}, compared with the Max-Log-MAP iterative demapper, but it offers a performance gain of 0.3 dB over the noniterative Max-Log-MAP demapper. Clearly, for transmission over the AWGN channel, the proposed simplified iterative demapper only suffers from a slight performance loss in comparison with the high-complexity optimal Max-Log-MAP iterative demapper.

The BER results obtained for transmission over the Rayleigh fading channel are shown in Fig. 6. It is shown that, at the BER level of 10^{-4}, our reduced-complexity iterative demapper suffers from performance losses of 0.1 and 0.25 dB for the 64QAM and 256QAM, respectively, compared with the standard Max-Log-MAP iterative demapper. By contrast, it offers performance gains of 0.42 and 0.33 dB over the noniterative Max-Log-MAP demapper for the 64QAM and 256QAM, respectively. Again, we can conclude that over the Rayleigh fading channel, the reduced-complexity iterative demapper only suffers from a slight performance loss compared with the Max-Log-MAP iterative demapper.

V. Conclusion

We have proposed a reduced-complexity iterative demapper suitable for the BICM-ID system relying on Gray-labeled constellation schemes. By exploiting the equivalent symbol that naturally includes the *a priori* LLR information of the soft demapper and the symmetry of Gray-labeled constellation, the calculation of the *a posteriori* LLR information is significantly simplified. Specifically, the computational complexity of the proposed simplified iterative demapper is on the order of $\mathsf{O}(m)$, in comparison with the $\mathsf{O}\left(2^m\right)$ complexity of the standard Max-Log-MAP iterative demapper, for the 2^m-ary constellation. Furthermore, the simulation results, obtained for both the 64QAM and 256QAM schemes for transmission over the AWGN and Rayleigh fading channels, confirm that the proposed reduced-complexity iterative demapper with the aid of an original, efficient, and low-complexity compensation scheme only suffers from a slight performance loss, as compared with the high-complexity optimal Max-Log-MAP iterative demapper.

In this paper, we have restricted our investigation to single-input–single-output frequency-nonselective channels. In our future investigation, we will consider how to extend the proposed reduced-complexity iterative demapper to frequency-selective channels and/or multiple-input–multiple-output systems.

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A Four-Element Array of Wideband Low-Profile H-Plane Horns

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***Abstract*—A wideband wide-coverage H-plane horn array with vertical polarization is proposed. Due to its light weight and low profile, the designed array is suitable for applications in spectrum monitoring systems on moving platforms. The switched-beam symmetrical array consists of four identical short H-plane horns. Pattern deterioration occurring in a high-frequency range is solved by inserting two pins in each horn. The measured impedance bandwidth is almost 100% for VSWR < 2 owing to its fed structure with a thick-cylinder-loaded probe. The measured azimuth-plane beamwidth is about 90° over the entire frequency range from 860 to 2500 MHz. Moreover, the measured results with the corresponding simulated results are presented and discussed in this paper.**

***Index Terms*—H-plane horn, spectrum monitoring system, vehicular application, wideband, wide-coverage.**

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I. Introduction

THE rapid growth of wireless communications introduces more frequency bands into application [1]. Thus, spectrum monitoring work, which can cover a wide frequency range and wide coverage, is required. This work is usually carried out on vehicles. Vertical polarization helps reduce the attenuation loss due to the ground plane of the Earth. Therefore, the antenna mounted on a car roof with vertical polarization is a good choice.

Numerous antennas designed for wideband automotive applications have been presented in the literature. L-probe-fed stacked patches with a novel design of grounded structure achieved an H-plane beamwidth over 90° and low backlobe radiation in [2]. However, the antenna radiates in broadside. Printed inverted-F antennas have been popular for multiband telematics applications on vehicles [3]–[5], and their impedance bandwidths are not sufficiently large for wideband spectrum monitoring systems. Monopoles of various shapes can achieve ultrawide impedance bands [6], [7]. Similar monopoles are designed for vehicular applications [8], [9]. However, they are not good candidates due to low gain and directivity.

Multisector antennas using a dedicated antenna for each sector can simply fulfill both requirements of coverage and high gain [10]–[12]. Monopole and slot Yagi-Uda arrays achieved a high gain of more than 10 dBi in [10] and [11]. A four-sector Vivaldi antenna was designed to achieve a 50% bandwidth and a 6-dBi gain in [12]. Nevertheless, there is no suitable design of multisector antennas with wide bandwidth, high gain, and low profile that can be readily mounted on a moving platform.

To achieve a wideband and wide-coverage antenna mounted on a car roof with vertical polarization, an H-plane horn fed by a thick-cylinder-loaded probe is presented in this paper. The radiation pattern of a regular H-plane horn has serious deterioration in the H-plane at high frequency due to the influence of higher order modes. Similar pattern deteriorations occur in other wideband aperture antennas such as double-ridged horns [13]. To solve this problem, methods such as modifications of the ridges, flares, and back cavity, which can influence the higher order modes, were proposed [14]–[16]. Furthermore, a direct method of using a higher order mode suppressor was adopted in [17]. These methods are carried out on ridge horns and not suitable for a regular H-plane horn. In this paper, a simple and effective method of inserting pins in each H-plane horn is employed to suppress the higher order modes and then improve the H-plane radiation patterns.

II. Design of the Proposed Antenna

A. Basic Principle

The structure of the presented array is shown in Fig. 1. The array consists of four identical H-plane horns placed in the azimuth plane. Due to its symmetry, the array scans beams with the same radiation patterns. Each H-plane horn is constructed from two side plates, back, upper, and down walls and has a rectangle radiated aperture with dimensions of $a \times b$. The designed horn is fed by a thick-cylinder-loaded probe, and this fed structure acts as a $\lambda/4$ monopole [18]–[20]. Hence, the directivity D of the horn can be roughly estimated using the half-power beamwidth of the H-plane radiation pattern $\theta_{0.5}$. The horn on a large ground plane can be thought to radiate in a beam with a beamwidth of $\theta_{0.5}$ in the azimuth plane. After comparing with a $\lambda/4$ monopole on an infinite ground plane, the following relationship can be derived:

$$D = \frac{2\pi}{\theta_{0.5}} D_m \tag{1}$$