

Channel Code-Division Multiple Access and Its Multilevel-Structured LDPC-Based Instantiation

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Abstract—In this paper, we introduce and outline the concept of channel code-division multiple access using a design example based on the recently proposed multilevel-structured (MLS) low-density parity-check (LDPC) codes. We succeeded in making the memory requirements of the multiuser transceiver to become practically independent of the total number of users supported by the system, as well as ascertain that each user benefits from the same level of protection. Finally, we will demonstrate that despite their beneficial compact structure, the proposed MLS LDPC codes do not suffer from any bit-error-ratio or block-error-ratio performance degradation, when compared to an otherwise identical benchmarker scheme using significantly more complex LDPC codes having pseudorandom parity-check matrices.

Index Terms—Channel code-division multiple access (CCDMA), multilevel-structured (MLS) low-density parity-check (LDPC) codes.

I. INTRODUCTION

The concept of generalized code-division multiple access (CDMA) may be defined as a multiple-access scheme that separates the users in the code domain while allowing them to share the same time and frequency resources. A traditional way of generating the user-specific codes is by employing distinct spreading codes, as in the well-known direct-sequence CDMA (DS-CDMA) [1] scheme. Another possibility is to distinguish between users using user-specific channel codes, which is reminiscent of the concept of trellis-coded multiple access (TCMA) [2] and interleave-division multiple access (IDMA) [3]. In the former, the separation of the users is achieved by the unique combination of user-specific generator polynomials (GPs) combined with bit-to-symbol mapping schemes and interleavers, while the latter employs user-specific interleavers, which may be regarded as rate-one channel codes. Hence, we will jointly refer to these schemes using the generic terminology of channel CDMA (CCDMA).¹

Typically, a relatively short code constraint length is favored in TCMA systems to attain a reasonably low decoding complexity. Naturally, this reduces both the number of GPs and the number of users supported. It reduces the probability of encountering randomlike low-correlation codewords. For this reason, it is widely recognized that in a TCMA system, a user-specific interleaver is required at the output of the trellis-coded modulation (TCM) scheme to achieve good bit-error-

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¹On a practical note, CCDMA may be employed for differentiating several users or symbol streams transmitted within the same time or frequency slot or users sharing the same DS-CDMA sequence. In this sense, it has a philosophy similar to spatial-division multiplexing, where the users/streams are differentiated by their unique impulse responses.

ratio (BER) performance [2], since the interleaved codewords become more randomlike and potentially impose a reduced interference owing to their lower correlation.² Consequently, a TCMA system can be considered to be a special case of IDMA [3], employing TCM codes as the outer channel code and dispensing with the direct-sequence-spreading stage of IDMA.

On the other hand, IDMA can be viewed as chip-interleaved DS-CDMA that employs binary transmitted symbols for each user and is applicable for low-rate uplink (UL) communications. It was also shown in [3] that the amalgamation of channel codes with IDMA systems further enhanced by sophisticated power allocation is capable of approaching the channel's capacity. Thus, it becomes evident that the family of pseudorandom LDPC codes such as those proposed in [4] and [5] constitutes particularly attractive component codes for CCDMA schemes, since they exhibit a near-capacity performance, as well as being capable of differentiating the users, with the aid of their inherent interleavers. Despite these advantages, LDPC-code-aided CCDMA may suffer from two potential drawbacks.

- 1) **Memory inefficiency:** Each user transmitting over the Q -user multiple-access channel (MAC) is encoded as well as decoded by a channel code having a distinct parity-check matrix (PCM). This implies that a different PCM must be stored in a lookup table (LUT) for each user.³ Therefore, the memory requirements are (linearly) dependent on both the LDPC code's block length and on the PCM parameters, as well as on the number of users supported by the system.
- 2) **Unequal protection:** When using LDPC codes having pseudorandom PCMs, it becomes quite difficult to construct a sufficiently high number of user-specific codes having identical graph-theoretical properties such as the girth.⁴

In this paper, we consider a CCDMA system based on the recently proposed multilevel-structured (MLS) LDPC codes [9]. Our novel contributions are the following.

- We circumvent the first problem of high memory requirements by taking advantage of the compact PCM description of MLS LDPC codes, which were recently proposed in [9].
- We ensure that each user's bits are equally protected, which is achieved by utilizing isotopic user-specific (distinct) Latin squares.⁵

To elaborate a little further, a J -level MLS code inherently possesses both pseudorandom and structured characteristics and can be described by means of a base matrix, J number of constituent matrices, and a so-called adjacency matrix, where the latter can be represented

²This is due to the fact that the cardinality of the set of interleaved codewords $|\Pi(\mathbb{C})|$ becomes significantly larger than that of $|\mathbb{C}|$, where \mathbb{C} represents the set of all possible codewords, and Π represents the set of user-specific interleavers.

³As an example, if each of the Q PCMs has a column weight of γ and a block length of N , then the LUT has to store the position of $QN\gamma$ nonzero PCM entries, each representing an edge of the corresponding Tanner graph [6].

⁴It is widely recognized that the performance of an LDPC code is quite dependent on the girth of the corresponding LDPC bipartite graph (refer to [7] and [8]). For example, the complexity of choosing pseudorandom LDPC codes having the same girth will become dependent on the variance of the girth average distribution [8] to maintain the same protection for each user.

⁵As one of the anonymous reviewers had pointed out, having identical average power for all the users does not mean that their interuser interference will be identical, because this depends on the correlation between the nonorthogonal pairs of channel code/interleaver combinations. This correlation is predetermined by the choice of the specific Latin squares used.

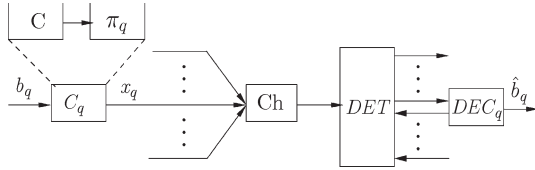


Fig. 1. General simplified model for a purely channel-coded IDMA-like CCDMA system.

by a Latin square. We will demonstrate that by using the same J constituent matrices for each user, the memory requirements become practically independent of the total number of users present in the system, since the users may be uniquely identified with the aid of a user-specific ($J \times J$)-element Latin square. Furthermore, by exploiting isotopic Latin squares, we propose a technique of constructing channel codes that are distinct while guaranteeing similar attainable BER/Block Error Ratio (BLER) performance for each user. We will demonstrate that despite imposing a beneficial compact structure, no BER or BLER performance degradation was imposed compared to the corresponding CCDMA system using pseudorandom LDPC codes for differentiating the users, albeit the latter imposes high memory requirements.

The structure of this paper is outlined as follows. Section II introduces the general model of the CCDMA system, while Section III outlines the construction of MLS codes. Then, Section IV details the technique proposed for generating user-specific channel codes by exploiting the construction of MLS LDPC codes. Our simulation results are then presented in Section V. Finally, our conclusions are offered in Section VI.

II. GENERAL MODEL OF THE CCDMA SYSTEM

Fig. 1 depicts the general model of the CCDMA system, where the q th user's signal b_q is encoded by his/her user-specific channel code C_q , $q = 1, \dots, Q$, having a rate of R , resulting in the codeword $x_q = C_q(b_q)$. In a conventional IDMA system, the channel code may be the same for all users if a user-specific interleaver is employed; hence, user q will transmit the bitstream of $x_q = \pi_q[C(b_q)]$ over the MAC. The canonical discrete-time real-valued model of the MAC seen in Fig. 1 is then given by

$$y = \sum_{q=1}^Q h_q x_q + n \quad (1)$$

where $x_q \in \{\pm 1\}$, y , and $n \sim \mathcal{N}(0, \sigma_n^2)$ denote the transmitted signal, the received signal, and the additive white Gaussian noise (AWGN), respectively. The parameter h_q denotes the identical independently distributed UL channel impulse response of user q , while σ_n^2 represents the noise variance.

An iterative receiver, consisting of a soft-in-soft-out (SISO) detector and a bank of Q individual SISO MLS LDPC decoders, is used for the sake of seeking a tradeoff between the higher performance and complexity of optimal joint detection and decoding and the performance loss of the lower complexity separate detection and single-user LDPC decoding. Using the low-complexity parallel interference cancellation (PIC) scheme introduced in [3], we can rewrite (1) as $y = h_q x_q + \xi$, where $\xi = \sum_{j \neq q} h_j x_j + n$ represents the interference plus noise. In the case of binary modulation, the real part (Re) of $h_q^* y$ constitutes sufficient statistics for estimating x_q , resulting in $Re(h_q^* y) = |h_q|^2 x_q + Re(h_q^* \xi)$, where $(\cdot)^*$ denotes the complex con-

jugate computation. We denote the soft estimate of a variable a by (\hat{a}) . Then, $Re(h_q^* \hat{\xi})$ and its variance $\mathbf{V}[Re(h_q^* \hat{\xi})]$ are formulated by

$$Re(h_q^* \hat{\xi}) = h_q^{Re} \hat{y}^{Re} + h_q^{Im} \hat{y}^{Im} - |h_q|^2 \hat{x}_q \quad (2)$$

$$\mathbf{V}[Re(h_q^* \hat{\xi})] = (h_q^{Re})^2 \mathbf{V}(\hat{y}^{Re}) + (h_q^{Im})^2 \mathbf{V}(\hat{y}^{Im}) - |h_q|^4 \mathbf{V}(\hat{x}_q) + 2h_q^{Re} h_q^{Im} \phi \quad (3)$$

where $\phi = \sum_{q=1}^Q h_q^{Re} h_q^{Im} \mathbf{V}(\hat{x}_q)$, and $Im(\cdot)$ represents the imaginary part of a complex number. The soft estimate \hat{y} and its variance can be expressed by

$$\hat{y}^{Re} = \sum_{q=1}^Q h_q^{Re} \hat{x}_q \quad (4)$$

$$\mathbf{V}(\hat{y}^{Re}) = \sum_{q=1}^Q (h_q^{Re})^2 \mathbf{V}(\hat{x}_q) + \sigma_n^2. \quad (5)$$

We remark that (4) and (5) would be equally valid for the imaginary counterpart. The soft bit \hat{x}_q can be represented as $\hat{x}_q = \tanh[L_{dec}^e(x_q)/2]$, while its variance is given by $\mathbf{V}(\hat{x}_q) = 1 - \hat{x}_q^2$. Assuming that ξ is Gaussian distributed, the extrinsic information $L_{det}^e(x_q)$ is given by

$$L_{det}^e(x_q) = 2|h_q|^2 \frac{Re(h_q^* y) - Re(h_q^* \hat{\xi})}{\mathbf{V}[Re(h_q^* \hat{\xi})]}. \quad (6)$$

Then, this extrinsic information gleaned from the detector is used as the *a priori* information input to the channel decoder, which computes a more reliable extrinsic information $L_{dec}^e(x_q)$ for the next iteration. LDPC decoding was performed using the sum-product algorithm (SPA) [10], where messages are exchanged between the nodes residing at both sides of the corresponding Tanner graph.

III. CONSTRUCTION OF MLS LDPC CODES AND THEIR LATIN SQUARE REPRESENTATION

We consider MLS LDPC codes described by their regular bipartite graphs $G(\mathbf{H})$, associated with a PCM \mathbf{H} whose rows constitute the null space of the linear code \mathbb{C} constructed over $\text{GF}(2)$. Then, the graph $G(\mathbf{H})$ consists of the nonempty set of elements $\{V(G), C(G), E(G)\}$, where $V(G) = v_1, v_2, \dots, v_N$ and $C(G) = c_1, c_2, \dots, c_M$ represent the disjoint vertex sets of the variable nodes and check nodes, while $E(G)$ is the set representing the edges. Furthermore, we assume that the degree of the variable nodes $v_r \in V(G)$, $r \in [1, N]$, is γ , and that of the check nodes $c_s \in C(G)$, $s \in [1, M]$, is ρ . Since the PCM of an MLS code \mathbf{H} has a full rank and a size of $(M \times N)$ elements, where the MLS LDPC code's block length is $N = |V(G)|$, the code rate becomes $R = 1 - M/N$. Each nonzero entry in \mathbf{H} will then represent an edge of the corresponding Tanner graph.

The construction of a J -level MLS code requires a base matrix, a set of J constituent matrices, and an adjacency matrix [9]. The base matrix, hereby denoted by \mathbf{H}^b , is a sparse matrix defined over $\text{GF}(2)$ having $(M^b \times N^b)$ elements and contains exactly ρ and γ nonzero entries, randomly positioned in each of its rows and columns, respectively. The set of J constituent matrices is then represented by $\Omega = \{\mathbf{Q}_0, \mathbf{Q}_1, \dots, \mathbf{Q}_{J-1}\}$, where the nonzero constituent matrix \mathbf{Q}_j , $j = 0, \dots, J-1$, is a distinct sparse matrix over $\text{GF}(2)$ having the

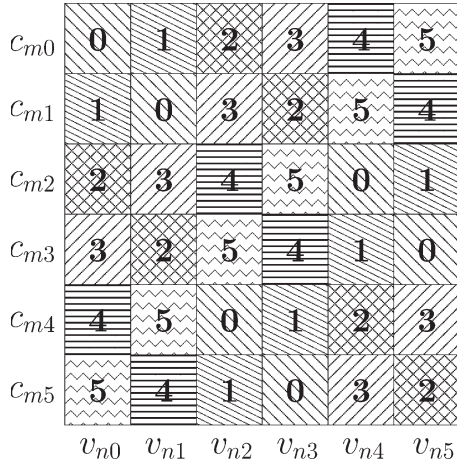


Fig. 2. Latin square representation of the adjacency matrix of a six-level MLS code. The J rows and columns of the Latin square correspond to the respective multicheck node c_{mj} and multivariable node v_{nj} , where $m = 1, \dots, M^b$, $n = 1, \dots, N^b$, and $j = 0, \dots, J - 1$.

same dimensions as the base matrix. All the nonzero entries of all the J sparse constituent matrices in $\mathbf{Q}_j \in \Omega$ must occur in the same corresponding position of the nonzero entries of the base matrix.⁶ Furthermore, a nonzero entry in a particular location of $\mathbf{Q}_j \in \Omega$ implies that the entries in the corresponding locations of $\mathbf{Q}_i \in \Omega$ are zero, where $i \in [0, J - 1]$, and $i \neq j$.

Finally, we also define an adjacency matrix, which is a $(J \times J)$ -element array matrix represented by \mathbf{P}_J , whose row (and column) blocks represent a sharply transitive set of J permutations within Ω . This implies that given any pair of constituent matrices $\mathbf{Q}_x, \mathbf{Q}_y \in \Omega$, there exists a unique bijective⁷ mapping function $f : \Omega \mapsto \Omega$ in the set described by the row (and column) block of \mathbf{P}_J that maps $\mathbf{Q}_x \in \Omega$ to the image $\mathbf{Q}_y = f(\mathbf{Q}_x) \in \Omega$. The adjacency matrix determines the position of each $\mathbf{Q}_j \in \Omega$ with respect to the PCM \mathbf{H} of the MLS code. From its definition, it follows that the adjacency matrix will not position any constituent matrix in the same row or column block to achieve the required check and variable node distribution while at the same time ensuring that the girth of the corresponding Tanner graph is at least six.

From another point of view, it can be argued that the J rows and columns of \mathbf{P}_J represent a Latin square of order J or, equivalently, a 1-factorization of a bipartite graph.⁸ Consequently, a J -level MLS code can also be regarded as an edge-colored complete bipartite graph of degree J . This brings us to the notion of what is known as coloring [12] of edges, where $E(\mathbf{H})$ is said to be an edge coloring of $G(\mathbf{H})$ if any two edges on the graph containing the same vertex have different colors. Fig. 2 depicts the Latin square representation for a six-level MLS code, where in this case, its adjacency matrix is effectively a reduced Latin square. The J rows and columns of the Latin square will then correspond to the respective multicheck node $c_{mj} \subset C(\mathbf{H})$ and to the multivariable node $v_{nj} \subset V(\mathbf{H})$, $|c_{mj}| = |v_{nj}|$, where we have $m = 1, \dots, M^b$, $n = 1, \dots, N^b$, and $j = 0, \dots, J - 1$. Each symbol

⁶Symmetrically repeated nonzero entries in two or more rows (or columns) in any of the matrices $\mathbf{Q}_j \in \Omega$, $j = 0, \dots, J - 1$, can be avoided by ensuring that the base matrix corresponds to a base protograph (refer to [9]) having a girth of at least six. Note furthermore that the position of the nonzero entries in each of the constituent matrices in $\mathbf{Q}_j \in \Omega$ is also chosen at random.

⁷The function $f : X \mapsto Y$ is said to be a bijective function (or simply a bijection) if every $y \in Y$ is an $f(x)$ value for exactly one $x \in X$.

⁸The interested reader is referred to [11] for a proof of this theorem.

in the Latin square will then correspond to a *multiedge* with a different edge color on the corresponding graph.

IV. GENERATING USER-SPECIFIC CHANNEL CODES USING MLS LDPC CODES

In this light, two seemingly contradictory problems must be outlined. First, since the Q users are being separated in the MLS LDPC code domain, a user-specific channel code is required. This makes the memory requirements at the transceiver dependent on the number of users present in the system, which is undesirable in memory-constrained handheld transceivers. Second, each of the Q users must be guaranteed the same BER/BLER performance at any SNR. Therefore, the channel code must be distinct while at the same time guaranteeing similar attainable BER/BLER performance for each user. These two problems are tackled separately in the forthcoming sections.

A. User Separation by Distinct Latin Squares

We reduce the memory requirements by using the same base matrix and J constituent matrices for all the Q users in the system. This implies that the receiver will only have J distinct memory blocks, each corresponding to a constituent matrix having a dimension that is a factor of $1/J$ lower than that of a single PCM. However, a distinct adjacency matrix is then allocated for each user, and hence, the required user separation is achieved by assigning a different Latin square to each of the Q users. The number of distinct Latin squares of order J is given by $\mathcal{X}_J = J! \times (J - 1)! \times L(J, J)$ [13], where $L(J, J)$ is the number of normalized $(J \times J)$ -element Latin squares.

For the sake of simplifying our analysis, let us consider the simple example of a six-level MLS code. The total number of Latin squares of order six is equal to $\mathcal{X}_6 = 6! \times 5! \times 9408$ [13]. This means that using a six-level MLS code, we can describe a total of \mathcal{X}_6 unique PCMs, corresponding to \mathcal{X}_6 unique Tanner graphs and thus representing a total of \mathcal{X}_6 unique binary codes, while still sharing the same base matrix and requiring a total of only six constituent matrices for differentiating the Q users. Therefore, a CCDMA system employing six-level MLS codes can potentially distinguish between a total of \mathcal{X}_6 users by only storing six $(M^b \times N^b)$ -element constituent matrices and Q adjacency matrices, where we have $M^b = M/J$ and $N^b = N/J$. The dimension of an adjacency matrix is only $(J \times J)$, where J is much smaller than both M and N ; therefore, its storage requirements can be considered negligible when compared to the $(M \times N)$ -element PCM. Therefore, our proposed system renders the memory requirements practically independent of the total number of users supported by the system. On the other hand, any other LDPC-code-aided CCDMA system has to store Q PCMs, each having a dimension of $(M \times N)$, thus requiring in total the enumeration of $QN\gamma$ number of edges.

B. Isotopic Latin Squares and Isomorphic Edge-Colored Graphs

This section outlines the technique that was employed to ensure that all the Q users benefit from the same level of protection. This brings us to the notion of isotopic Latin squares and isomorphic edge-colored graphs. Two Latin squares S and S' are said to be isotopic if there exists a triple (α, β, χ) (referred to as an isotopy,⁹ where α, β , and χ correspond to a row, column, and symbol permutation, respectively, which carries the Latin square S to S' . Effectively, this implies that

⁹Then, an isotopy class comprises the set of all the Latin squares isotopic to a given Latin square. A list of the isotopy classes for Latin squares of small orders is given by McKay in [14].

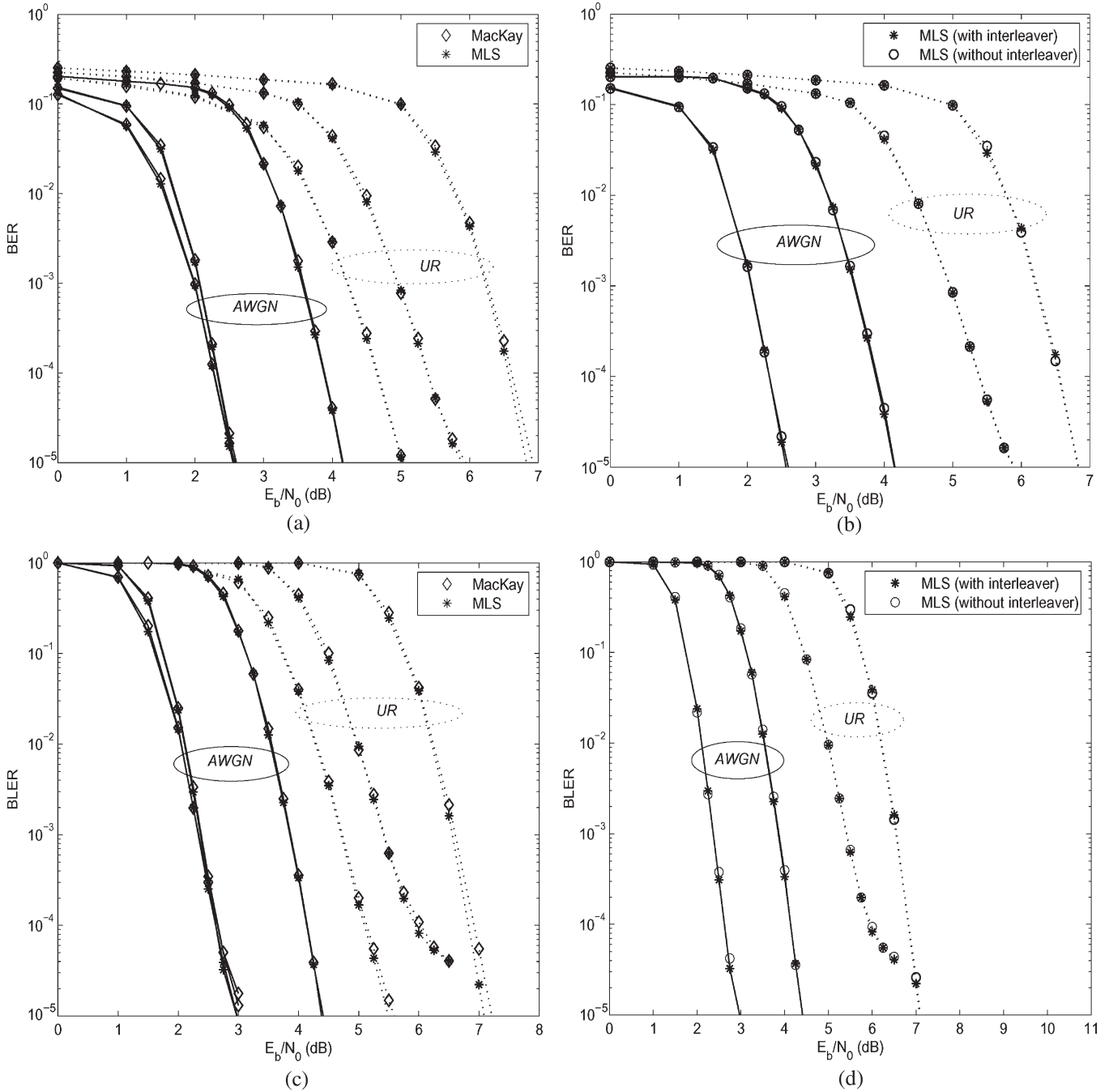


Fig. 3. BER and BLER performance comparison of a purely channel-coded IDMA using half-rate MacKay [16] and six-level MLS LDPC codes having a block length of N when transmitting over the AWGN and UR channels. (a) and (b) Comparison of the BER and BLER performance of the IDMA system using MacKay and MLS codes, where both systems also have a user-specific interleaver, for one, two, and three users. (c) and (d) Comparison of the BER and BLER performance for two and three users of the IDMA system using MLS codes with and without the user-specific interleaver.

if we consider any particular row and column position of the Latin square specified by the check nodes and variable nodes (c_{mj}, v_{nj}) and containing the entry e , where $m = 1, \dots, M^b$, $n = 1, \dots, N^b$, and $j = 0, \dots, J - 1$, the entry at position $(\alpha(c_{mj}), \beta(v_{nj}))$ of the Latin square S' will be equal to $\chi(e)$. Two Latin squares S_1 and S_2 will then give rise to isomorphic edge-colored complete bipartite graphs if and only if Latin square S_1 is isotopic to either another Latin square S_2 or to its transpose [15].

Since the decoding of LDPC codes is very much dependent on their graph-theoretic properties, we can ensure the same level of protection for each user if all the user-specific channel codes C_q ,

$q = 1, \dots, Q$, have the corresponding edge-colored Tanner graphs that exhibit identical graph-theoretic properties and thus are isomorphic. This can be achieved by allocating adjacency matrices to the Q users that are represented by both distinct and isotopic Latin squares.

V. SIMULATION RESULTS

The results presented in this section were obtained using BPSK modulation, when transmitting over the AWGN and uncorrelated Rayleigh (UR) MACs and using LDPC-code-aided CCDMA systems in conjunction with both six-level MLS codes [9] and pseudorandom

MacKay [16] codes. We have considered half-rate LDPC codes having $\gamma = 3$ and a block length of $N = 1008$ bits. The number of users supported by the system was $Q = 3$, and therefore, the bandwidth efficiency defined as RQ was 1.5 b/s/Hz. The number of iterations between the PIC detector and the LDPC decoder was set to $I = 5$ for $Q = 2$ users and $I = 10$ for $Q = 3$ users. The LDPC decoding was performed using the SPA having a maximum of 100 iterations.

Fig. 3 illustrates the BER and BLER performance comparison of a CCDMA scheme using half-rate MacKay and six-level MLS LDPC codes having a block length of $N = 1008$, when transmitting over the AWGN and UR channels. In Fig. 3(a) and (b), we compare the achievable BER as well as the BLER performance using the $N = 1008$ MacKay and MLS codes as component codes and a user-specific pseudorandom interleaver after the channel encoder. It can be observed that the BER/BLER performance of both systems is comparable. We point out that in this case, there is no need for a distinct code description C_q for each user, $q \in [1, Q]$, since the LDPC-encoded bitstream of each user is interleaved by a user-specific interleaver before being transmitted over the MAC. Our motivation of showing the results in Fig. 3(a) and (b) is to explicitly demonstrate that both systems have similar performance.

We then proceed to remove the user-specific interleaver, when user separation is entirely achieved by the distinct (and isotopic) Latin squares. The BER and BLER performance exhibited in this scenario is shown in Fig. 3(c) and (d), where we compared the performance of the MLS LDPC-coded CCDMA system both with and without the interleaver. Once again, the proposed system does not suffer from any BER/BLER performance loss.

However, the proposed system has considerable gains in terms of the interleaver storage and delay requirements, since there is no need to store user-specific interleavers or a user-specific PCM. For the case of the benchmarker system using the pseudorandom MacKay codes, the memory LUT must store the location of 9072 edges to fully describe the three distinct PCMs. On the other hand, the IDMA system using MLS LDPC codes as component codes is more memory efficient, since in this case, the LUT has to enumerate only 612 edges to store the six distinct (84×168)-element constituent matrices and the three (6×6)-component Latin squares (adjacency matrices). Furthermore, we note that the difference in the memory requirements of the two systems will become more pronounced upon increasing the number of users Q or the block length N . The proposed system will be applicable in situations where low-delay requirements are an absolute necessity, for example, in interactive lip-synchronized speech and video communications.

VI. SUMMARY AND CONCLUSION

The concept of CCDMA has been proposed, arguing that the application of an LDPC-code-aided CCDMA system in memory-constrained scenarios may become cumbersome, since a distinct PCM code description is required for each user, which has to be stored in memory, to be able to differentiate each user. In this paper, we have proposed a specific instantiation of a CCDMA system using MLS LDPC codes, where we exploited the compactness of the MLS LDPC code description to significantly reduce the memory requirements. By using the same J constituent matrices for each user, we have succeeded in rendering the memory requirements practically independent of the total number of users present in the system, since each user is only distinguished by means of a different ($J \times J$)-component Latin square instead of a different PCM. Furthermore, we have outlined a technique based on isotopic Latin squares that makes it possible to construct channel codes that are distinct while guaranteeing a

similar attainable BER/BLER performance for each user. We have demonstrated that these advantages accrue without any compromise in the attainable BER/BLER performance, when compared to the corresponding pseudorandom-LDPC-based CCDMA benchmarker, which imposes significantly higher memory requirements. Our scheme is attractive in interactive low-delay speech and video applications and is equally applicable for other classes of randomlike codes such as repeat-accumulate codes.

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Fully detailed technical reports on this research are available to Industrial Members of Mobile VCE.

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