Minimum Bit Error Rate Multiuser Transmission Designs Using Particle Swarm Optimisation

W. Yao, S. Chen, S. Tan, and L. Hanzo

***Abstract*—We consider the downlink of a multiuser system equipped with multiple antennas transmitting to multiple single-antenna mobile receivers. Particle swarm optimisation (PSO) is invoked to solve the constrained nonlinear optimisation problem for the minimum bit error rate (MBER) multiuser transmitter (MUT). The proposed PSO aided symbol-specific MBER-MUT and average MBER-MUT schemes provide improved performance in comparison to the conventional minimum mean-square-error MUT scheme, while imposing a reduced complexity compared to the state-of-the-art sequential quadratic programming based symbol-specific MBER-MUT and average MBER-MUT schemes, respectively.**

***Index Terms*—Downlink, multiuser transmission, precoding, minimum mean square error, minimum bit error rate, sequential quadratic programming, particle swarm optimisation.**

I. Introduction

In the downlink (DL) of space-division multiple-access (SDMA) system, non-cooperative mobile stations (MSs) are unable to employ multiuser detection (MUD). In order to facilitate the employment of low-complexity single-user-receiver, the transmitted signals have to be preprocessed at the base station (BS), leading to the appealing concept of multiuser transmission (MUT) [1], provided that accurate channel state information (CSI) is available at the transmitter. The assumption that the DL channel impulse response (CIR) is known at the BS may be deemed valid in time division duplex (TDD) systems, where the uplink (UL) and DL signals are transmitted at the same frequency, provided that the co-channel interference is similar at the BS and MSs which is not always the case. MUT-aided transmit preprocessing may hence be deemed attractive, when the channel's coherence time is longer than the transmission burst interval. However, for frequency division duplex systems, where the UL and DL channels are expected to be different, CIR feedback from the MS's receivers to the BS transmitter is necessary [2].

In general, if the BS could not estimate the UL CIR matrix and therefore could not use the UL CIR matrix as the DL CIR matrix in MUT, the MS's receivers have to signal their DL CIRs to the BS transmitter via the UL channels [2]. Alternatively, blind channel estimation can be adopted at the cost of increased complexity [3]. Precoding can also be employed to mitigate self interference caused by space division multiplexing, as in the work [4], where the transmitter employs two antennas while each receiver also has two antennas to provide multiplexing gain. A simple zero-forcing precoding scheme is used at the transmitter in [4] for each user to combat self interference caused by multiplexing[1].

The MUT design based on the minimum mean-square-error (MMSE) criterion is popular owing to its appealing simplicity [2], [5]. However, since the bit error rate (BER) is the ultimate system performance indicator, research in minimum BER (MBER) based MUT techniques has intensified recently. A MBER-MUT scheme was proposed in [6] for the TDD code-division multiple-access DL designed for frequency-selective channels and this work was extended to multiple transmit and receive antennas in [7]. A chip-level MBER-MUT scheme was proposed in [8]. The precoder's weight matrix of the above-mentioned MBER-MUT techniques [6]–[8] is calculated specifically for each transmit symbol vector. Since the coefficients of the MUT have to be recalculated individually for every transmit symbol vector, we refer to this type of MBER-MUT techniques as the symbol-specific MBER-MUT scheme. To mitigate the complexity imposed, an average MBER-MUT design was proposed for both binary phase shift keying (BPSK) [9] and quadrature phase shift keying (QPSK) [10], where the coefficients of the precoder only have to be recalculated when the channel coefficients change substantially. All the MBER-MUT designs invoke a constrained nonlinear optimisation [9]–[11] and, as a state-of-the-art, the sequential quadratic programming (SQP) algorithm [12] is typically used to obtain the precoder's coefficients [9]–[11]. However, the computational complexity of the SQP based MBER-MUT solution may be excessive for high-rate systems [11].

Hence, as an attractive design alternative, in this contribution we invoke the particle swarm optimisation (PSO) algorithm [13] to find the MBER-MUT's coefficients in order to reduce its complexity. PSO constitutes a population based stochastic optimisation technique [14], which is inspired by the social behaviour of bird flocks or fish schools. The algorithm commences with random initialisation of a swarm of individuals, referred to as particles, within the specific problem's search space. It then endeavours to find a globally optimum solution by gradually adjusting the trajectory of each particle towards its own best location and towards the best position of the entire swarm at each evolutionary optimisation step. The PSO method is popular owing to its simplicity in implementation, ability to rapidly converge to a "reasonably good" solution and to "steer clear" of local minima. It has been successfully applied to wide-ranging optimisation problems

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[1]The scheme of [4] is not a MUT, as precoding is not used for mitigating multiuser interference. User separation is provided by code spreading, i.e. CDMA.

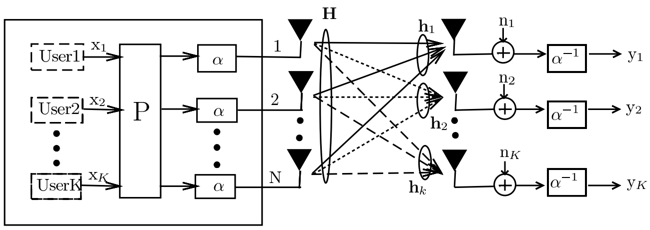

Fig. 1. Schematic diagram of the downlink of a SDMA system using preprocessing at the BS. The MUT-aided system employs N DL transmit antennas to communicate with K non-cooperative mobile devices.

[15]–[17]. More particularly, many researchers have applied PSO techniques to multiuser detection (MUD) [18]–[21]. However, to the best of our knowledge, no work to date has applied the PSO method to MUT design. We will show that the proposed PSO approach is capable of finding the optimal MBER MUT solution at a lower complexity than the benchmark state-of-the-art SQP based MBER MUT design.

Throughout our discussions we adopt the following notational conventions. Boldface capitals and lower-case letters stand for matrices and vectors, respectively. The (p,q)th element $h_{p,q}$ of $\mathbf{H}$ is also denoted by $\mathbf{H}|_{p,q}$. Furthermore, $(\,)^T$ represents the transpose operator, while $\|\,\|^2$ and $|\,|$ denote the norm and the magnitude operators, respectively. $E[\,]$ denotes the expectation operator, while $\Re[\,]$ and $\Im[\,]$ represent the real and imaginary parts, respectively. Finally, $j=\sqrt{-1}$.

II. System Model

The DL of a SDMA system considered is depicted in Fig. 1. The BS is equipped with N transmit antennas and communicates over flat fading channels with K MSs, each employing a single-receive antenna. Frequency selective channels can be converted to a multiplicity of parallel narrowband channels using for example the orthogonal frequency division multiplexing technique [22]. In the system model of Fig. 1, the vector of information symbols transmitted in the DL is given by $\mathbf{x}=[x_1\ x_2\cdots x_K]^T$, where x_k denotes the transmitted symbol to the kth MS and the symbol energy is given by $E[|x_k|^2]=\sigma_x^2$, for $1\le k\le K$. The MUT's $(N\times K)$-element precoder matrix $\mathbf{P}$ is defined by

$$\mathbf{P}=[\mathbf{p}_1\ \mathbf{p}_2\cdots\mathbf{p}_K], \quad (1)$$

where $\mathbf{p}_k$, $1\le k\le K$, is the precoder's coefficient vector for the kth user's data stream. Given a fixed total transmit power $\mathrm{E_T}$ at the BS, an appropriate scaling factor should be used to fullfill this transmit power constraint, which is defined as

$$\alpha=\sqrt{\mathrm{E_T}/E[\|\mathbf{Px}\|^2]}. \quad (2)$$

At the receiver, the reciprocal of the scaling factor, namely α^{-1}, is used to scale the received signal to ensure unity-gain transmission. The channel matrix $\mathbf{H}$ is given by

$$\mathbf{H}=[\mathbf{h}_1\ \mathbf{h}_2\cdots\mathbf{h}_K], \quad (3)$$

where $\mathbf{h}_k=[h_{1,k}\ h_{2,k}\cdots h_{N,k}]^T$, $1\le k\le K$, is the kth user's spatial signature. The channel taps $h_{i,k}$ for $1\le k\le K$ and $1\le i\le N$ are independent of each other and obey the complex-valued Gaussian distribution associated with $E[|h_{i,k}|^2]=1$. The additive white Gaussian noise vector $\mathbf{n}$ is defined by $\mathbf{n}=[n_1\ n_2\cdots n_K]^T$, where n_k, $1\le k\le K$, is a complex-valued Gaussian random process with zero mean and $E[|n_k|^2]=2\sigma_n^2=\mathrm{N_o}$. The signal-to-noise ratio (SNR) of the DL is defined as $\mathrm{SNR}=\mathrm{E_b}/\mathrm{N_o}$, where $\mathrm{E_b}=\mathrm{E_T}/(N\log_2 M)$ is the energy per bit per antenna for M-ary modulation. Thus, the baseband model of the system can be described as

$$\mathbf{y}=\mathbf{H}^T\mathbf{Px}+\alpha^{-1}\mathbf{n}, \quad (4)$$

where $\mathbf{y}=[y_1\ y_2\cdots y_K]^T$ denotes the received signal vector, and y_k, $1\le k\le K$, constitutes sufficient statistics for the kth MS to detect the transmitted data symbol x_k. Thus, the kth MS equipped with a conventional matched filter can simply estimate x_k by quantising y_k.

III. MBER Multiuser Transmission

For notational simplicity, we consider the QPSK modulation with $M=4$. Extension to the generic quadrature amplitude modulation can be achieved by considering the minimum symbol error rate criterion, as in the MUD case [23]. Two MBER-MUT design strategies exist in the literature. The first design [11] may be referred to as the symbol-specific MBER MUT, while the other one [9], [10] can be termed as the average MBER MUT.

A. Symbol-Specific MBER-MUT

This approach is developed based on the fact that the information symbols to be transmitted are known exactly at the transmitter, and hence the precoding matrix can be chosen specifically for the given QPSK symbol vector so that the probability of error or BER is minimised for this specific transmission of symbol vector [11]. Given the DL QPSK symbol vector $\mathbf{x}$, the average BER of the in-phase component of $\mathbf{y}$ at the receivers is [7]

$$P_{e_I,\mathbf{x}}=\frac{1}{K}\sum_{k=1}^{K}Q\left(\frac{\mathrm{sgn}(\Re[x_k])\Re[\mathbf{h}_k^T\mathbf{Px}]}{\sigma_n}\right), \quad (5)$$

where $Q(\bullet)$ is the standard Gaussian error function. Similarly, given the symbol vector $\mathbf{x}$, the average BER of the quadrature-phase component of $\mathbf{y}$ is [7]

$$P_{e_Q,\mathbf{x}}=\frac{1}{K}\sum_{k=1}^{K}Q\left(\frac{\mathrm{sgn}(\Im[x_k])\Im[\mathbf{h}_k^T\mathbf{Px}]}{\sigma_n}\right). \quad (6)$$

Thus, the resultant BER for the specific QPSK symbol $\mathbf{x}$ is

$$P_{e,\mathbf{x}}(\mathbf{P})=\big(P_{e_I,\mathbf{x}}(\mathbf{P})+P_{e_Q,\mathbf{x}}(\mathbf{P})\big)/2. \quad (7)$$

Therefore, the symbol-specific MBER-MUT design is defined as the solution of the following constrained optimisation

$$\begin{aligned}\mathbf{P}_{\text{TxMBER},\mathbf{x}} &= \arg\min_{\mathbf{P}} P_{e,\mathbf{x}}(\mathbf{P}) \\ \text{s.t.}\quad & \|\mathbf{Px}\|^2=\mathrm{E_T}.\end{aligned} \quad (8)$$

Since the precoder's matrix $\mathbf{P}_{\text{TxMBER},\mathbf{x}}$ depends on the symbol vector $\mathbf{x}$ to be transmitted, it must be recalculated for each transmitted symbol vector. To alleviate the complexity imposed by this symbol-specific MBER MUT design, the alternative average MBER MUT design chooses the precoder matrix that remains optimal for all the legitimate transmission symbol vectors.

TABLE I
COMPUTATIONAL COMPLEXITY PER ITERATION OF FOUR MBER MUT DESIGNS FOR QPSK SIGNALLING, WHERE N IS THE NUMBER OF TRANSMIT ANTENNAS, K THE NUMBER OF MOBILE USERS, $M = 4$ IS THE SIZE OF SYMBOL CONSTELLATION AND S IS THE PARTICLE SIZE.

Algorithm	Flops
SQP(Symbol-specific MBER MUT)	$\mathcal{O}(8 \times K^3 \times N^3) + 8 \times K^3 \times N^2 + 8 \times N^2 \times K^2 + 22 \times K^2 \times N$ $+8 \times K \times N^2 + 14 \times K^2 + 18 \times K \times N - 2 \times N^2 + 2 \times K + N + 11$
PSO(Symbol-specific MBER MUT)	$(38 \times N \times K + 8 \times N + 7 \times K + 3) \times S + 8$
SQP(Average MBER MUT)	$K \times (8 \times N^2 \times K^2 + 6 \times N \times K + 6 \times N + 8 \times K + 4) \times M^K$ $+\mathcal{O}(8 \times N^3 \times K^3) + 8 \times N^2 \times K^2 + 16 \times N \times K^2 + 8 \times N^2 \times K$ $+12 \times N \times K + 6 \times K^2 - 2 \times N^2 + N - 2 \times K + 11$
PSO(Average MBER MUT)	$((16 \times N \times K + 7 \times K + 6 \times N + 1) \times M^K + 20 \times N \times K + 2) \times S + 8$

B. Average MBER-MUT

The average BER of the in-phase component of $\mathbf{y}$ at the receivers can be shown to be [24]

$$P_{e_I} = \frac{1}{KM^K} \sum_{q=1}^{M^K} \sum_{k=1}^{K} Q\left(\frac{\text{sgn}(\Re[x_k^{(q)}])\Re[\mathbf{h}_k^T \mathbf{P}\mathbf{x}^{(q)}]}{\sigma_n}\right). \quad (9)$$

Here $M^K = 4^K$ is the number of equiprobable legitimate transmit symbol vectors $\mathbf{x}^{(q)}$ for QPSK signalling and $x_k^{(q)}$ the kth element of $\mathbf{x}^{(q)}$, with $1 \leq q \leq M^K$. Similarly, the average BER of the quadrature-phase component of $\mathbf{y}$ is given by [24]

$$P_{e_Q} = \frac{1}{KM^K} \sum_{q=1}^{M^K} \sum_{k=1}^{K} Q\left(\frac{\text{sgn}(\Im[x_k^{(q)}])\Im[\mathbf{h}_k^T \mathbf{P}\mathbf{x}^{(q)}]}{\sigma_n}\right). \quad (10)$$

Thus the average BER for QPSK signalling is given by

$$P_e(\mathbf{P}) = \big(P_{e_I}(\mathbf{P}) + P_{e_Q}(\mathbf{P})\big)/2, \quad (11)$$

and the solution of the average MBER MUT is defined as

$$\begin{aligned} \mathbf{P}_{\text{TxMBER}} &= \arg\min_{\mathbf{P}} P_e(\mathbf{P}) \quad (12) \\ \text{s.t.} \quad & E[\|\mathbf{P}\mathbf{x}\|^2] = \text{E}_\text{T}. \end{aligned}$$

The problems (8) and (12) are constrained nonlinear optimisation ones, and they are typically solved by an iterative gradient based algorithm known as the SQP [9]–[11]. The complexities per iteration of the SQP based symbol-specific MBER MUT and average MBER MUT schemes, respectively, are listed in Table I for QPSK modulation, where $\mathcal{O}(\bullet)$ stands for order of $\bullet$ complexity and we assume that the complexity of a real-valued multiplication is equal to a real-valued addition. The complexity per optimisation equals the number of iterations that the algorithm required to arrive at a global optimal solution multiplied by this complexity per iteration. For the symbol-specific MBER-MUT, the precoding matrix needs to be recalculated for every transmit symbol vector and hence it exhibits much higher computational complexity in the long run. On the other hand, it provides a slightly better performance in comparison with the average MBER-MUT design.

IV. PSO Assisted MBER MUT

A penalty function approach is adopted to convert the constrained optimisation processes (8) and (12) into the unconstrained ones and to automatically perform power allocation in order to meet the transmit power constraint.

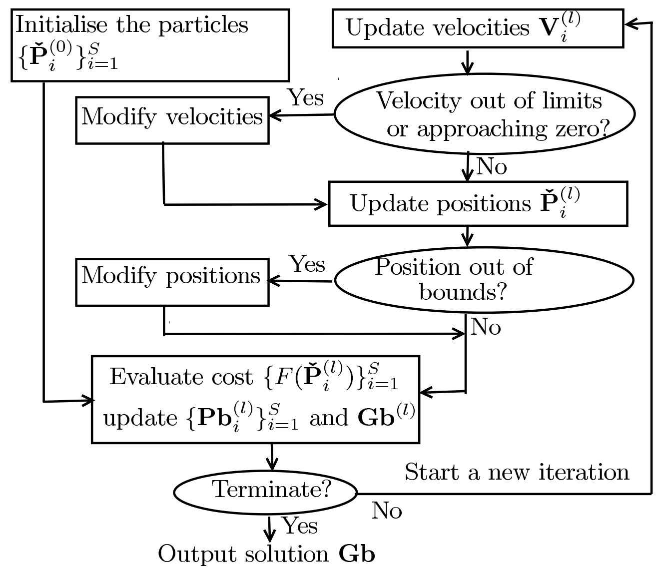

Fig. 2. Flowchart of the particle swarm optimisation algorithm.

For the *symbol-specific MBER-MUT*, we define the cost function

$$F(\mathbf{P}) = P_{e,\mathbf{x}}(\mathbf{P}) + G_{\mathbf{x}}(\mathbf{P}) \quad (13)$$

with the penalty function given by

$$G_{\mathbf{x}}(\mathbf{P}) = \begin{cases} 0, & \|\mathbf{P}\mathbf{x}\|^2 - \text{E}_\text{T} \leq 0, \\ \lambda(\|\mathbf{P}\mathbf{x}\|^2 - \text{E}_\text{T}), & \|\mathbf{P}\mathbf{x}\|^2 - \text{E}_\text{T} > 0, \end{cases} \quad (14)$$

where the penalty factor $\lambda > 0$ should be chosen appropriately so that the symbol-specific MBER-MUT design (8) becomes the following unconstrained optimisation

$$\mathbf{P}_{\text{TxMBER},\mathbf{x}} = \arg\min_{\mathbf{P}} \{P_{e,\mathbf{x}}(\mathbf{P}) + G_{\mathbf{x}}(\mathbf{P})\}. \quad (15)$$

For the *average MBER-MUT*, we define the cost function

$$F(\mathbf{P}) = P_e(\mathbf{P}) + G(\mathbf{P}) \quad (16)$$

with the penalty function given by

$$G(\mathbf{P}) = \begin{cases} 0, & E[\|\mathbf{P}\mathbf{x}\|^2] - \text{E}_\text{T} \leq 0, \\ \lambda(E[\|\mathbf{P}\mathbf{x}\|^2] - \text{E}_\text{T}), & E[\|\mathbf{P}\mathbf{x}\|^2] - \text{E}_\text{T} > 0. \end{cases} \quad (17)$$

With an appropriately chosen penalty factor λ, the average MBER-MUT design (12) can be obtained as the solution of the following unconstrained optimisation

$$\mathbf{P}_{\text{TxMBER}} = \arg\min_{\mathbf{P}} \{P_e(\mathbf{P}) + G(\mathbf{P})\}. \quad (18)$$

The flowchart of the PSO aided MBER-MUT design is given in Fig. 2. A swarm of particles, $\{\check{\mathbf{P}}_i^{(l)}\}_{i=1}^S$, that represent potential solutions are evolved in the search space $\mathsf{S}^{N\times K}$, where

$$\mathsf{S} = [-P_{\max},\ P_{\max}] + j[-P_{\max},\ P_{\max}] \quad (19)$$

is the square area in the complex plan that defines the search range for each element of the precoder coefficient matrix, S is the swarm size and index l denotes the iteration step.

a) The swarm initialisation. With $l = 0$, set $\check{\mathbf{P}}_1^{(l)}$ to the MMSE solution, while the rest of the initial particles, $\{\check{\mathbf{P}}_i^{(l)}\}_{i=2}^S$, are randomly generated in the search space $\mathsf{S}^{N\times K}$.

b) The swarm evaluation. Each particle $\check{\mathbf{P}}_i^{(l)}$ has a cost $F(\check{\mathbf{P}}_i^{(l)})$ associated with it, which is evaluated according to either (13) or (16), depending on whether the symbol-specific or average MBER-MUT design is considered. Each particle $\check{\mathbf{P}}_i^{(l)}$ remembers its best position visited so far, denoted as $\mathbf{Pb}_i^{(l)}$, which provides the cognitive information. Every particle also knows the best position visited so far among the entire swarm, denoted as $\mathbf{Gb}^{(l)}$, which provides the social information. $\{\mathbf{Pb}_i^{(l)}\}_{i=1}^S$ and $\mathbf{Gb}^{(l)}$ are updated at each iteration:

For $(i = 1;\ i \leq S;\ i{+}{+})$
  If $(F(\check{\mathbf{P}}_i^{(l)}) < F(\mathbf{Pb}_i^{(l)}))$ $\quad \mathbf{Pb}_i^{(l)} = \check{\mathbf{P}}_i^{(l)}$;
End for;
$i^* = \arg\min_{1\leq i\leq S} F(\mathbf{Pb}_i^{(l)})$;
If $(F(\mathbf{Pb}_{i^*}^{(l)}) < F(\mathbf{Gb}^{(l)}))$ $\quad \mathbf{Gb}^{(l)} = \mathbf{Pb}_{i^*}^{(l)}$;

c) The swarm update. Each particle $\check{\mathbf{P}}_i^{(l)}$ has a velocity, denoted as $\mathbf{V}_i^{(l)}$, to direct its "flying". The velocity and position of the ith particle are updated in each iteration according to

$$\begin{aligned}\mathbf{V}_i^{(l+1)} &= w * \mathbf{V}_i^{(l)} + rand() * c_1 * (\mathbf{Pb}_i^{(l)} - \check{\mathbf{P}}_i^{(l)}) \\ &\quad + rand() * c_2 * (\mathbf{Gb}^{(l)} - \check{\mathbf{P}}_i^{(l)}), \quad (20)\\ \check{\mathbf{P}}_i^{(l+1)} &= \check{\mathbf{P}}_i^{(l)} + \mathbf{V}_i^{(l+1)}, \quad (21)\end{aligned}$$

where w is the inertia weight, $rand()$ denotes the uniform random number between 0 and 1, and c_1 and c_2 are the two acceleration coefficients. In order to avoid excessive roaming of particles beyond the search space [16], a velocity range, $\mathsf{V} = [-V_{\max},\ V_{\max}] + j[-V_{\max},\ V_{\max}]$, is imposed on each element of $\mathbf{V}_i^{(l+1)}$ so that

If $(\Re[\mathbf{V}_i^{(l+1)}|_{p,q}] > V_{\max})$ $\quad \Re[\mathbf{V}_i^{(l+1)}|_{p,q}] = V_{\max}$;
If $(\Re[\mathbf{V}_i^{(l+1)}|_{p,q}] < -V_{\max})$ $\quad \Re[\mathbf{V}_i^{(l+1)}|_{p,q}] = -V_{\max}$;
If $(\Im[\mathbf{V}_i^{(l+1)}|_{p,q}] > V_{\max})$ $\quad \Im[\mathbf{V}_i^{(l+1)}|_{p,q}] = V_{\max}$;
If $(\Im[\mathbf{V}_i^{(l+1)}|_{p,q}] < -V_{\max})$ $\quad \Im[\mathbf{V}_i^{(l+1)}|_{p,q}] = -V_{\max}$;

Moreover, if the velocity (20) approaches zero, it is reinitialised to proportional to $V_{\max}$ with a small factor γ

$$\mathbf{V}_i^{(l+1)}|_{p,q} = \pm rand() * \gamma * (V_{\max} + jV_{\max}). \quad (22)$$

Similarly, each element of $\check{\mathbf{P}}_i^{(l+1)}$ is checked to ensure that it stays inside the search space S:

If $(\Re[\check{\mathbf{P}}_i^{(l+1)}|_{p,q}] > P_{\max})$
  $\Re[\check{\mathbf{P}}_i^{(l+1)}|_{p,q}] = rand() * P_{\max}$;
If $(\Re[\check{\mathbf{P}}_i^{(l+1)}|_{p,q}] < -P_{\max})$
  $\Re[\check{\mathbf{P}}_i^{(l+1)}|_{p,q}] = -rand() * P_{\max}$;
If $(\Im[\check{\mathbf{P}}_i^{(l+1)}|_{p,q}] > P_{\max})$
  $\Im[\check{\mathbf{P}}_i^{(l+1)}|_{p,q}] = rand() * P_{\max}$;
If $(\Im[\check{\mathbf{P}}_i^{(l+1)}|_{p,q}] < -P_{\max})$
  $\Im[\check{\mathbf{P}}_i^{(l+1)}|_{p,q}] = -rand() * P_{\max}$;

That is, if a particle is outside the search space, it is moved back inside the search space randomly, rather than forcing it to stay at the border. This is similar to the scheme given in [16].

d) Termination condition check. If the maximum number of iterations, $I_{\max}$, is reached, terminate the algorithm with the solution $\mathbf{Gb}^{(I_{\max})}$; otherwise, set $l = l + 1$ and go to Step *b)*.

It was reported in [15] that using a time varying acceleration coefficient (TVAC) enhances the performance of PSO. We adopt this mechanism, in which c_1 is reduced from 2.5 to 0.5 and c_2 varies from 0.5 to 2.5 during the iterative procedure according to

$$\begin{aligned}c_1 &= (0.5 - 2.5) * l/I_{\max} + 2.5,\\ c_2 &= (2.5 - 0.5) * l/I_{\max} + 0.5.\end{aligned} \quad (23)$$

This TVAC mechanism works well in our application. We also remove the influence of the previous velocity by setting $w = 0$, as suggested in [15]. Our empirical results suggest that the search limit can be set to $P_{\max} = 1$ for our application. With this choice of position bound, the velocity limit can be set to $V_{\max} = 1$. An appropriate value of the small control factor γ in (22) for avoiding zero velocity is found to be $\gamma = 0.1$ by experiments. The computational complexity per iteration for the PSO based symbol-specific MBER-MUT and average MBER-MUT schemes, respectively, are also listed in Table I.

V. Simulation Study

The DL of the multiuser system considered employed $N = 4$ transmit antennas at the BS to communicate over the (4×4)-element flat Rayleigh fading channels to $K = 4$ single-receive-antenna MSs. The swarm size was chosen to be $S = 20$, which was found empirically to be appropriate for our PSO-aided MBER-MUT design problems. The maximum number of iterations, $I_{\max}$, was so chosen such that the PSO-based MBER-MUT algorithm with the chosen $I_{\max}$ and $S = 20$ arrived at the same MBER performance also achieved by the SQP-based MBER-MUT design. It was found empirically that an adequate $I_{\max}$ was in the range of 20 to 40, depending on the value of the channel SNR. All the simulation results were obtained by averaging over 100 channel realisations.

Fig. 3 compares the BER performance of the MMSE-MUT scheme with those of the PSO-based symbol-specific MBER-MUT and average MBER-MUT schemes, assuming a perfect CSI knowledge at the BS. It is seen that the PSO aided symbol-specific MBER-MUT achieved an SNR gain of 4.5 dB over the MMSE-MUT at the target BER of 10^{-4}, while the PSO aided average MBER-MUT provided an SNR gain of 3 dB over the MMSE-MUT scheme at the same target BER level. The robustness of the two PSO-aided MBER-MUT schemes to channel estimation error was also investigated by adding a Gaussian white noise with a standard deviation of 0.05 per dimension to each channel tap $h_{i,k}$ to represent channel estimation error. The BERs of the MMSE-MUT and the two PSO based MBER-MUT schemes under this channel

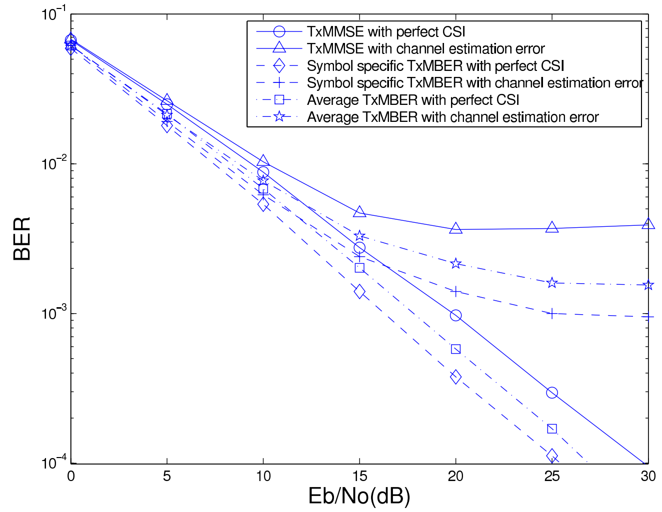

Fig. 3. BER versus SNR performance of the PSO aided symbol-specific MBER-MUT and average MBER-MUT communicating over flat Rayleigh fading channels using $N = 4$ transmit antennas to support $K = 4$ QPSK mobile users, in comparison with the benchmark MMSE-MUT.

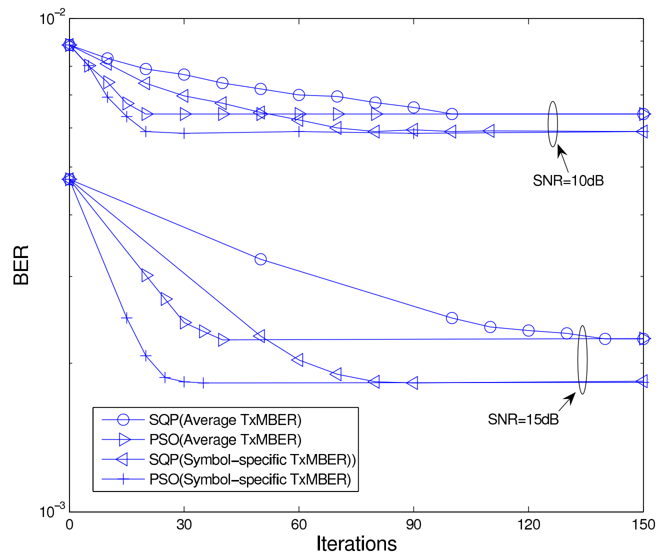

Fig. 4. Convergence performance of the SQP as well as PSO aided symbol-specific and average MBER-MUT schemes for the system employing $N = 4$ transmit antennas to support $K = 4$ QPSK mobile users over flat Rayleigh fading channels at Eb/No=10 dB and 15 dB, repsectively.

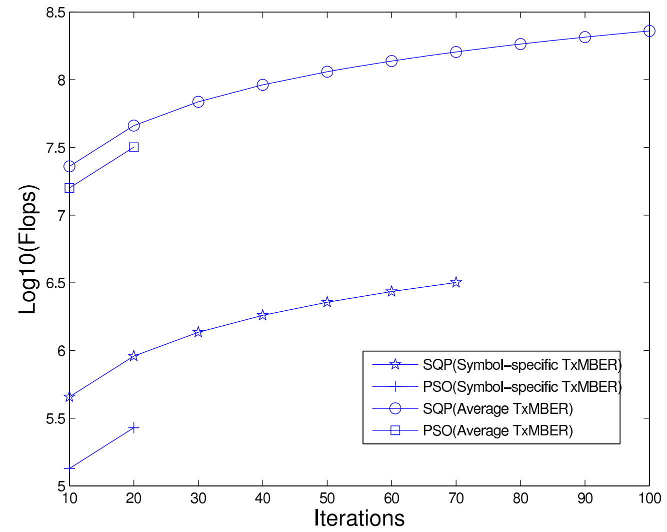

Fig. 5. Complexity per optimisation comparison of the SQP as well as PSO aided symbol-specific and average MBER-MUT schemes for the system employing $N = 4$ transmit antennas to support $K = 4$ QPSK mobile users over flat Rayleigh fading channels at Eb/No=10 dB.

estimation error are also plotted in Fig. 3. It can be seen that both the symbol-specific and average MBER-MUT designs were no more sensitive to channel estimation error than the MMSE-MUT design. Fig. 4 compares the convergence performance of the SQP as well as PSO based symbol-specific and average MBER MUT schemes, operating at the SNR values of 10 dB and 15 dB, respectively.

For the symbol-specific MBER MUT design at SNR$=$ 10 dB, it can be seen from Fig. 4 that the SQP algorithm converged to the optimal solution after 70 iterations, while the PSO counterpart arrived at the same optimal solution after 20 iterations. In the case of average MBER-MUT design at SNR$=$ 10 dB, the SQP algorithm converged to the optimal solution after 100 iterations, while the PSO algorithm arrived at the same optimal solution after 20 iterations. Fig. 5 shows the complexity per optimisation of the four MBER-MUT designs studied at the SNR value of 10 dB. For the symbol-specific MBER MUT at SNR$=$ 10 dB, the SQP algorithm required 3,180,170 Flops to converge to the optimal solution, while the PSO aided algorithm converged to the same optimal solution at the cost of only 268,560 Flops. Hence the PSO-aided symbol-specific MBER-MUT design imposed an approximately twelve times lower complexity than the SQP counterpart for this case. For the average MBER MUT design at SNR$=$ 10 dB, the SQP algorithm needed 229,351,100 Flops to converge to the optimal solution, while the PSO aided algorithm converged to the same optimal solution at the cost of 34,561,760 Flops. Therefore, the PSO-aided average MBER-MUT design imposed an approximately seven times lower complexity than the SQP counterpart for this scenario. We also recorded the run times for the four MBER-MUT schemes, and the run-time saving achieved by a PSO based design over the SQP counterpart agreed with the above complexity analysis. Further investigation also showed that the convergence results for SNR$<$ 10 dB were similar to the case of SNR$=$ 10 dB.

From Fig. 4, it can also be seen that at SNR$=$ 15 dB the SQP based symbol-specific MBER MUT converged after 80 iterations which required 3,634,480 Flops, while the PSQ-aided symbol-specific MBER MUT achieved the convergence after 30 iterations which required 402,840 Flops. Thus, the PSO-aided symbol-specific MBER-MUT design imposed an approximately nine times lower complexity than the SQP counterpart for the SNR value of 15 dB. Similarly, it can be seen from Fig. 4 that at SNR$=$ 15 dB the SQP based average MBER MUT algorithm took 140 iterations to converge at a total cost of 321,091,540 Flops, while the PSO-aided average MBER MUT design needed 40 iterations to converge at a total cost of 63,541,120 Flops. It was then obvious that the PSO-aided average MBER-MUT design imposed an approximately five times lower complexity than the SQP counterpart for this scenario. The further convergence results obtained for SNR$>$ 15 dB were seen to be similar to the case of SNR$=$ 15 dB.

VI. Conclusions

We have proposed a PSO assisted MBER-MUT design, which offers a significantly lower complexity than the existing state-of-the-art SQP based MBER-MUT approach. Specifically, our simulation results involving the system of four transmit antennas and four QPSK mobile users over flat Rayleigh fading channels have confirmed that the proposed PSO-aided symbol-specific MBER-MUT scheme imposes approximately nine to twelve times lower complexity than the SQP-based symbol-specific MBER-MUT design, while the PSO-aided average MBER-MUT scheme imposes approximately five to seven times lower complexity than the SQP-based average MBER-MUT counterpart.

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