Differential Evolution Algorithm Aided MBER Beamforming Receiver for Quadrature Amplitude Modulation Systems

Jiankang Zhang[†‡], Sheng Chen[†], Xiaomin Mu[‡] and Lajos Hanzo[†]

[†] Communications, Signal Processing and Control Group
Electronics and Computer Science
Faculty of Physical and Applied Science
University of Southampton, Southampton SO17 1BJ, UK
E-mails: {jz09v,sqc,lh}@ecs.soton.ac.uk

[‡] School of Information Engineering
Zhengzhou University, Zhengzhou, China
E-mail: iexmmu@zzu.edu.cn

11th UK Workshop on Computational Intelligence
University of Manchester, Sept. 7-9, 2011

Outline

Outline

Background

- Many wireless communication system designs manifests as challenging global optimisation problems
  - Attaining **global** or near global **optimal solutions** at **affordable** computational **costs** are critical
- State-of-the-art minimum bit error rate beamforming receiver is such a design for
  - Multi-antenna wireless systems employing high throughput quadrature amplitude modulation
- We have successful record in applications of **computational intelligence** methods, such as
  - genetic algorithms, ant colony / particle swarm optimisation, differential evolution algorithm

DE Algorithm

- Key metrics in assessing a global optimisation method
  - **Capability**: high success rate to attain global solutions in challenging problems
  - **Complexity**: fast convergence speed and reasonably low computational costs
  - **Simplicity**: few algorithmic parameters need tuning and easy of programming
- Differential evolution algorithm measures well by these metrics
  - High success rate, reasonably fast convergence speed, and not too many tuning parameters
- This is what motivates us to develop DE algorithm aided MBER beamforming receiver
  - This work is pure application DE algorithm

Outline

System Model

- SDMA with L-element receive antenna array to support M QAM users, where receive signal vector $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_L(k)]^{\mathrm{T}}$

$$\mathbf{x}(k) = \mathbf{P}\mathbf{b}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k)$$

- Complex-valued AWGN vector $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots\ n_L(k)]^{\mathrm{T}}$ with covariance matrix $E[\mathbf{n}(k)\mathbf{n}^H(k)] = 2\sigma_n^2 \mathbf{I}_L$
- Channel matrix $\mathbf{P} = [A_1\mathbf{s}_1\ A_2\mathbf{s}_2 \cdots A_M\mathbf{s}_M] = [\mathbf{p}_1\ \mathbf{p}_2 \cdots \mathbf{p}_M]$ with ith channel coefficient A_i and steering vector for user i

$$\mathbf{s}_i = \left[e^{j\omega_c t_1(\theta_i)}\ e^{j\omega_c t_2(\theta_i)} \ldots e^{j\omega_c t_L(\theta_i)} \right]^{\mathrm{T}}$$

$t_l(\theta_i)$: relative time delay at array element l for user i, θ_i: direction of arrival for user i, $\omega_c = 2\pi f_c$: angular carrier frequency

- Transmitted symbol vector of M users $\mathbf{b}(k) = [b_1(k) \cdots b_M(k)]^{\mathrm{T}}$

Beamforming Receiver

- Assume **user 1** is desired user, beamformer output

$$y(k) = \mathbf{w}^{\mathrm{H}}\mathbf{x}(k) = \bar{y}(k) + e(k) = c_1 b_1(k) + \sum_{i=2}^{M} c_i b_i(k) + e(k)$$

$c_1 b_1(k)$: **desired signal**, summation term: **residual interfering signal**, $e(k)$: zero-mean **Gaussian** with $E[|e(k)|^2] = 2\sigma_n^2 \mathbf{w}^{\mathrm{H}}\mathbf{w}$

- Weight vector $\mathbf{w} = [w_1 \ w_2 \cdots w_L]^{\mathrm{T}}$, c_1 must be real and positive
- 16-QAM modulation, 4 bits per complex-valued symbol:

$$b_i(k) = b_{R_i}(k) + j b_{I_i}(k) \in \{\pm 1 \pm j, \ \pm 1 \pm 3j, \ \pm 3 \pm j, \ \pm 3 \pm 3j\}$$

- Two bits per in-phase / quadrature symbol mapping:

$$\mathbf{11}, \mathbf{10}, \mathbf{00}, \mathbf{01} \leftrightarrow -\mathbf{3}, -\mathbf{1}, +\mathbf{1}, +\mathbf{3}$$

Notice the class 1 (**C1**) bit and the class 2 (**C2**) bit

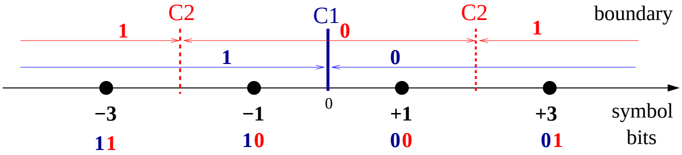
Detection of Bits

- $y(k) = y_R(k) + jy_I(k)$ used to detect four bits of $b_1(k)$
- Decision for in-phase C1 bit is given by

$$\begin{cases} \text{C1 bit} = 0, & \text{if } y_R(k) > 0 \\ \text{C1 bit} = 1, & \text{if } y_R(k) \leq 0 \end{cases}$$

and decision for in-phase C2 bit is given by

$$\begin{cases} \text{C2 bit} = 0, & \text{if } -2c_1 < y_R(k) < 2c_1 \\ \text{C2 bit} = 1, & \text{if } y_R(k) \leq -2c_1 \text{ or } y_R(k) \geq 2c_1 \end{cases}$$



- Decisions for quadrature C1 and C2 bits are given similarly based on $y_I(k)$

Outline

Bit Error Rate

- BER of 16-QAM beamformer with weight vector $\mathbf{w}$ is defined by

$$P_E(\mathbf{w}) = \frac{1}{4}\Big(P_{E_R,C1}(\mathbf{w}) + P_{E_I,C1}(\mathbf{w}) + P_{E_R,C2}(\mathbf{w}) + P_{E_I,C2}(\mathbf{w})\Big)$$

- Let $N_b = 16^M$, and $\mathbf{b}^{(q)}$, $1 \le q \le N_b$, be legitimate equiprobable sequences of $\mathbf{b}(k)$ $\Rightarrow$ Set of system vector states:

$$\bar{\mathbf{x}}(k) \in \mathbb{X} \triangleq \{\bar{\mathbf{x}}^{(q)} = \mathbf{P}\mathbf{b}^{(q)}, 1 \le q \le N_b\}$$

Set of beamformer scalar states

$$\bar{y}(k) \in \mathbb{Y} \triangleq \{\bar{y}^{(q)} = \mathbf{w}^{\mathrm{H}}\bar{\mathbf{x}}^{(q)}, 1 \le q \le N_b\} = \mathbb{Y}_R + j\mathbb{Y}_I$$

Conditional subsets, each having $N_{sb} = N_b/4$ points:

$$\begin{cases} \mathbb{Y}_R^{(\pm1)} \triangleq \{\bar{y}_R^{(q)} \in \mathbb{Y}_R : b_{R_1}(k) = \pm1\} \\ \mathbb{Y}_R^{(\pm3)} \triangleq \{\bar{y}_R^{(q)} \in \mathbb{Y}_R : b_{R_1}(k) = \pm3\} \end{cases}$$

C1 Bit Error Rate

- In-phase C1 bit error probability

$$P_{E_R,C1}(\mathbf{w}) = \frac{1}{2N_{sb}} \sum_{\bar{y}_R^{(q)} \in \mathbb{Y}_R^{(+)}} Q\left(g_{R,C1}^{(q)}(\mathbf{w})\right)$$

$$\mathbb{Y}_R^{(+)} = \mathbb{Y}_R^{(+1)} \bigcup \mathbb{Y}_R^{(+3)},\ Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-\frac{v^2}{2}}\, d\,v,$$

$$g_{R,C1}^{(q)}(\mathbf{w}) = \frac{\mathrm{sgn}(\Re[b_1^{(q)}])\bar{y}_R^{(q)}}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}} = \frac{\mathrm{sgn}(b_{R_1}^{(q)})\Re[\mathbf{w}^{\mathrm{H}}\bar{\mathbf{x}}^{(q)}]}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}}$$

- Quadrature C1 bit error probability

$$P_{E_I,C1}(\mathbf{w}) = \frac{1}{2N_{sb}} \sum_{\bar{y}_I^{(q)} \in \mathbb{Y}_I^{(+)}} Q\left(g_{I,C1}^{(q)}(\mathbf{w})\right)$$

$\mathbb{Y}_I^{(+)} = \mathbb{Y}_I^{(+1)} \bigcup \mathbb{Y}_I^{(+3)}$ and

$$g_{I,C1}^{(q)}(\mathbf{w}) = \frac{\mathrm{sgn}(\Im[b_1^{(q)}])\bar{y}_I^{(q)}}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}} = \frac{\mathrm{sgn}(b_{I_1}^{(q)})\Im[\mathbf{w}^{\mathrm{H}}\bar{\mathbf{x}}^{(q)}]}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}}$$

C2 Bit Error Rate

- In-phase C2 bit error probability $P_{E_R,C2}(\mathbf{w})$

$$P_{E_R,C2}(\mathbf{w}) = \frac{1}{2N_{sb}} \sum_{\bar{y}_R^{(q)} \in \mathbb{Y}_R^{(+1)}} \Big(Q(g_{R,C2}^{(q,a)}(\mathbf{w})) + Q(g_{R,C2}^{(q,b)}(\mathbf{w})) \Big) + \frac{1}{2N_{sb}} \sum_{\bar{y}_R^{(q)} \in \mathbb{Y}_R^{(+3)}} \Big(Q(g_{R,C2}^{(q,c)}(\mathbf{w})) - Q(g_{R,C2}^{(q,a)}(\mathbf{w})) \Big)$$

$$g_{R,C2}^{(q,a)}(\mathbf{w}) = \frac{2c_1 + \mathrm{sgn}(b_{R_1}^{(q)})\bar{y}_R^{(q)}}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}},\ g_{R,C2}^{(q,b)}(\mathbf{w}) = \frac{2c_1 - \mathrm{sgn}(b_{R_1}^{(q)})\bar{y}_R^{(q)}}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}},\ g_{R,C2}^{(q,c)}(\mathbf{w}) = \frac{\mathrm{sgn}(b_{R_1}^{(q)})\bar{y}_R^{(q)} - 2c_1}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}}$$

- Quadrature C2 bit error probability $P_{E_I,C2}(\mathbf{w})$

$$P_{E_I,C2}(\mathbf{w}) = \frac{1}{2N_{sb}} \sum_{\bar{y}_I^{(q)} \in \mathbb{Y}_I^{(+1)}} \Big(Q(g_{I,C2}^{(q,a)}(\mathbf{w})) + Q(g_{I,C2}^{(q,b)}(\mathbf{w})) \Big) + \frac{1}{2N_{sb}} \sum_{\bar{y}_I^{(q)} \in \mathbb{Y}_I^{(+3)}} \Big(Q(g_{I,C2}^{(q,c)}(\mathbf{w})) - Q(g_{I,C2}^{(q,a)}(\mathbf{w})) \Big)$$

$$g_{I,C2}^{(q,a)}(\mathbf{w}) = \frac{2c_1 + \mathrm{sgn}(b_{I_1}^{(q)})\bar{y}_I^{(q)}}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}},\ g_{I,C2}^{(q,b)}(\mathbf{w}) = \frac{2c_1 - \mathrm{sgn}(b_{I_1}^{(q)})\bar{y}_I^{(q)}}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}},\ g_{I,C2}^{(q,c)}(\mathbf{w}) = \frac{\mathrm{sgn}(b_{I_1}^{(q)})\bar{y}_I^{(q)} - 2c_1}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}}$$

MBER Solution

- MBER beamformer solution is defined as

$$\mathbf{w}_{\text{MBER}} = \arg\min_{\mathbf{w}} P_E(\mathbf{w})$$

- MBER beamformer design may be obtained based on a gradient-descent numerical optimisation
  1. Gradient of $P_E(\mathbf{w})$ requires extensive computation
  2. Slow convergence and local minima problem
- We use DE algorithm to solve this optimisation
  - DE is characterised by initialisation, mutation, re-combination and selection operations invoked for exploring the search space in an iterative procedure, until some termination criteria are met

Outline

DE Algorithm

1. **Initialisation**: Randomly generate initial population (generation $g = 1$) of **population size** P_s within search space

$$\hat{\mathbf{w}}_{g,p_s} = \left[\hat{w}_{g,p_s,1} \ \hat{w}_{g,p_s,2} \cdots \hat{w}_{g,p_s,L}\right]^{\mathrm{T}}, 1 \leq p_s \leq P_s$$

2. **Mutation**: Create mutant vector by combining three different, randomly chosen vectors

$$\hat{\mathbf{v}}_{g,i} = \hat{\mathbf{w}}_{g,r_1} + \gamma \left(\hat{\mathbf{w}}_{g,r_2} - \hat{\mathbf{w}}_{g,r_3}\right), 1 \leq i \leq P_s$$

   - $\gamma \in (0, 1]$: **scaling factor**

3. **Crossover**: Adopt uniform crossover to generate trial vector

$$\hat{u}_{g,i,l} = \begin{cases} \hat{v}_{g,i,l}, & rand_l(0,1) \leq C_r \text{ or } l = l_{rand}, \\ \hat{w}_{g,i,l}, & \text{otherwise}, \end{cases} \quad 1 \leq i \leq P_s$$

   - $C_r \in [0, 1]$: **crossover probability**
   - $rand_l(0, 1)$: uniform random number in $[0,\ 1)$
   - l_{rand}: uniform random integer in $\{1, 2, \cdots, L\}$

DE Algorithm (continue)

4. **Selection**: determine target vector $\hat{\mathbf{w}}_{g,p_s}$ or trial vector $\hat{\mathbf{u}}_{g,p_s}$ survives to next generation

$$\hat{\mathbf{w}}_{g+1,p_s} = \begin{cases} \hat{\mathbf{u}}_{g,p_s}, & P_E(\hat{\mathbf{u}}_{g,p_s}) \le P_E(\hat{\mathbf{w}}_{g,p_s}), \\ \hat{\mathbf{w}}_{g,p_s}, & \text{otherwise}, \end{cases} \quad 1 \le p_s \le P_s$$

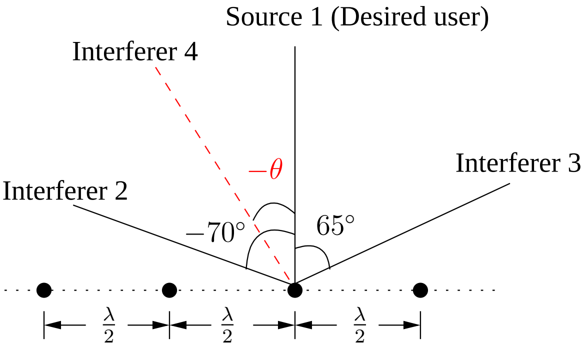
   based on cost function values

5. **Termination**: optimisation terminated when any of the following two stopping criteria are satisfied
   - Pre-defined maximum affordable number of generations G_{max} has been exhausted
   - Δg_{max} generations have been explored without a trial vector being accepted

Basic DE algorithm, with key algorithmic parameters **population size** P_s, **scaling factor** γ, and **crossover probability** C_r

Outline

Simulation Systems



1. **Full-rank**: four-element antenna array supporting four users
   - Minimum **angular separation** with desired user $\theta < 65°$
   - $\mathrm{E_b}/\mathrm{N_o}$: average bit energy over channel noise power
   - All channel taps A_i are identical
2. **Rank-deficient**: three-element array supporting four users

Benchmarks for Comparison

- Two beamforming receiver designs are used as benchmarks
  1. Conventional minimum mean square error (MMSE) solution that minimises MSE metric $E[|b_1(k) - y(k)|^2]$

     $$\mathbf{w}_{\text{MMSE}} = \left(\mathbf{P}\mathbf{P}^H + \frac{2\sigma_n^2}{\sigma_b^2}\mathbf{I}_L \right)^{-1} \mathbf{p}_1$$

     $2\sigma_n^2$: channel noise power, σ_b^2: average symbol power
  2. Our previous minimum symbol error rate (MSER) solution that minimises symbol error rate

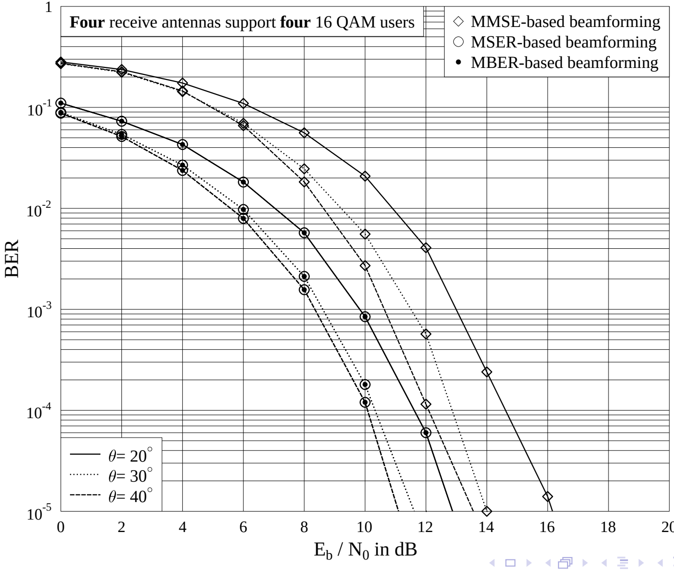
     $$\text{SER}(\mathbf{w}) = \text{Prob}\{\hat{b}_1(k) \neq b_1(k)\}$$

     $\hat{b}_1(k)$: detected symbol for $b_1(k)$
- Same DE algorithm used to obtain MSER solution

Outline

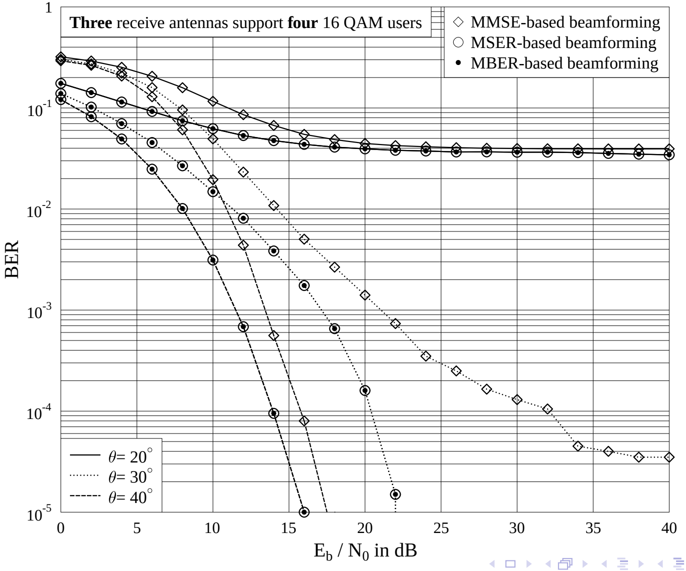
Bit Error Rate (full-rank)

- $P_s = 100$, $\gamma = 0.4$, $C_r = 0.4$, $G_{max} = 200$



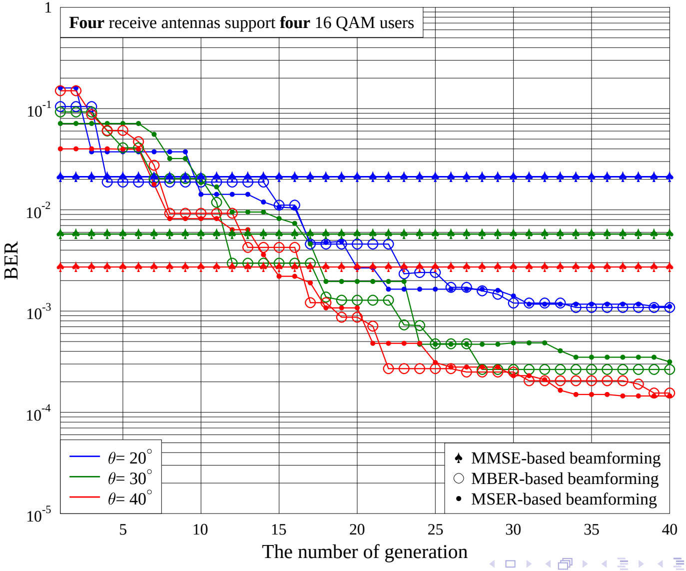
Bit Error Rate (rank-deficient)

- $P_s = 100$, $\gamma = 0.4$, $C_r = 0.4$, $G_{\max} = 200$



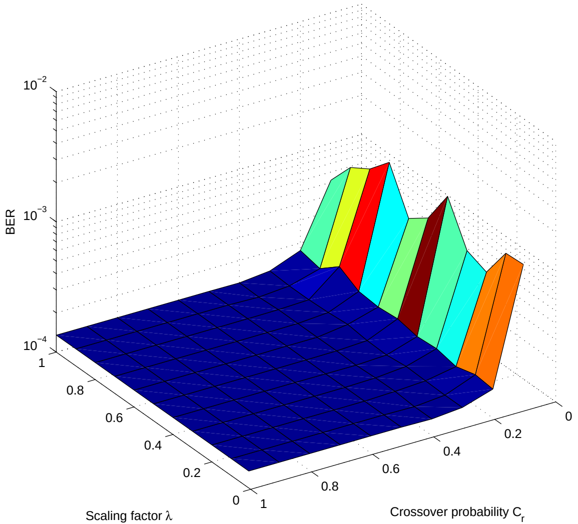
Convergence (full-rank)

- $P_s = 100$, $\gamma = 0.4$, $C_r = 0.4$, $E_b/n_0 = 10$ dB



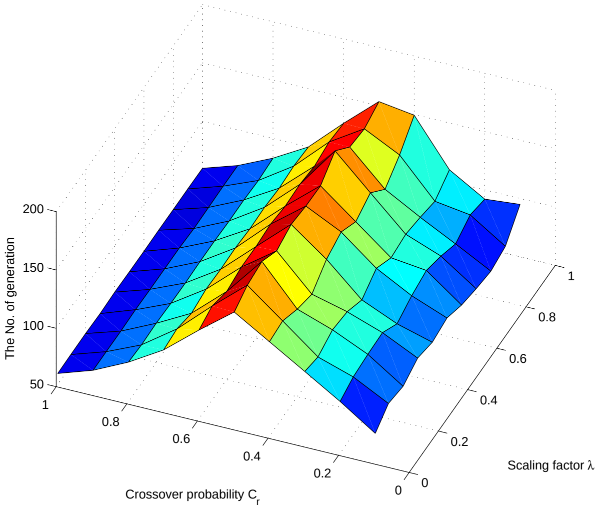
BER Sensitivity

- DE algorithm aided MBER: $P_S = 100$, $E_b/n_0 = 10$ dB



Convergence Sensitivity

- DE algorithm aided MBER: $P_S = 100$, $E_b/n_0 = 10$ dB



Outline

Summary

- We have proposed an DE algorithm aided MBER beamforming receiver for multi-user SDMA based QAM systems
  1. Derive explicitly BER expression
  2. Apply DE algorithm to both MBER and MSER designs
- Both MBER and MSER solutions significantly outperform MMSE design, in full-rank as well as rank-deficient scenarios
- Future work will compare GA, ACO, PSA and DE algorithm in benchmark communication system designs