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Stability issues of finite precision state estimate feedback controller realizations for discrete time systems[1]

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Abstract

This paper present a new effective algorithm for the optimal realization of state estimate feedback controller structures for discrete time systems subject to Finite-Word-Length (FWL) constraints. The problem is formulated as a nonlinear programming to provide an easy and efficient optimization tool to solve such complex problem. Simulation results of the optimum realizations of state estimate feedback controller structures are presented to illustrate the effectiveness of the proposed strategy.

Index Terms—Finite word length, stability, discrete time system.

1 Introduction

The recent advances in fixed-point implementation of digital controllers such as the design of dedicated fixed-point Digital Signal Processors (DSP) and new Digital Control Processors (DCP) architectures made Finite Word Length (FWL) implementation an important issue in modern digital control engineering design applications. Improved control performance and increased levels of integration are especially important in many areas such as consumer electronic products, automotive and electro-mechanical control systems. This is because hardware controller implementation with fixed-point arithmetic offer the advantages of speed, memory space, cost and simplicity over floating-point arithmetic.

The FWL effects have been well studied in digital signal processing, especially in digital filter implementation since the 1970's [1]. The results have recently been extended to the study of FWL effects of digital controller on control systems. [2] studied the effects of FWL implemented digital controller on the degradation of an LQG cost function from a statistical point of view. [3] analyzed the effects of FWL implemented digital controller on the stability and performance of sampled data systems. A FWL stability measure μ_0 was presented in [3], but computing explicity this measure seems very hard and is still an open problem. Based on the first order approximation, [4] and [5] developed two tractable FWL stability measures which are the lower bounds of μ_0 respectively.

In all these studies of FWL effects of digital controllers, the controllers are output feedback controllers.

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It is well known that there are another type of controllers, i.e. state-estimate feedback controllers. [6] studied the sensitivity and the roundoff noise gain of the closed-loop transfer function of a system when the state-estimate feedback controller is implemented with a FWL and when the computations are performed in finite precision. This paper intends to study the stability issues of finite precision state-estimate feedback controllers. One contribution of this paper is to compute the FWL stability measure for any realization of state-estimate feedback controllers. Another is to develop an algorithm of searching for the optimal state-estimate controller realization providing the maximal FWL stability measure.

2 Notation

Considering the discrete time closed-loop system with state-estimate feedback controller shown in Figure 1.

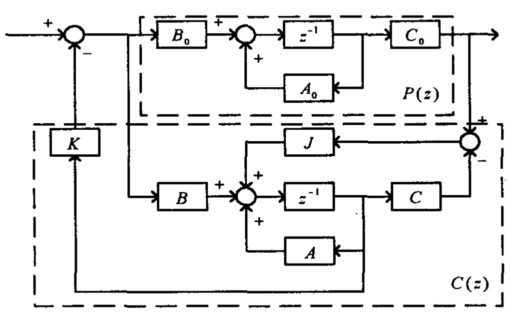

Figure 1. System with estimate feedback controller

where $P(z) = C_0(zI - A_0)^{-1}B_0$ is discrete time strictly proper plant, $A_0 \in R^{n\times n}, B_0 \in R^{n\times p}, C_0 \in R^{q\times n}$, $C(z)$ is discrete time state-estimate feedback controller. (A, B, C, K, J) is called a realization of $C(z)$. The realizations of $C(z)$ are not unique. In fact, all realizations of $C(z)$ can be discribed as

$$S_C = \{(A, B, C, K, J) \mid A = T^{-1}A_0T, B = T^{-1}B_0, \\ C = C_0T, K = K_0T, J = T^{-1}J_0\}. \quad (1)$$

The control gain $K_0 \in R^{p\times n}$ and the observer gain $J_0 \in R^{n\times q}$ have been given in state-estimate feedback controller design to make the closed loop system stable. Define U be the column stacking operator such that, for matrix X, $U(X)$ is a vector. Denote

$$w = \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix} = \begin{bmatrix} U(A) \\ U(B) \\ U(C) \\ U(K) \\ U(J) \end{bmatrix}, w_0 = \begin{bmatrix} w_{01} \\ \vdots \\ w_{0N} \end{bmatrix} = \begin{bmatrix} U(A_0) \\ U(B_0) \\ U(C_0) \\ U(K_0) \\ U(J_0) \end{bmatrix},$$

$N = n\times n + n\times p + q\times n + p\times n + n\times q$. Obviously, we can say realization w which has the same meaning as realization (A, B, C, K, J). For any realization w, the system matrix of the closed loop system is given by

$$\begin{aligned} \bar{A}(w) &= \begin{bmatrix} A_0 & -B_0K \\ JC_0 & A - BK - JC \end{bmatrix} \\ &= \begin{bmatrix} I_n & 0 \\ 0 & T^{-1} \end{bmatrix} \bar{A}(w_0) \begin{bmatrix} I_n & 0 \\ 0 & T \end{bmatrix} \end{aligned} \quad (2)$$

where I_n, I_p, I_q denote $n\times n, p\times p, q\times q$ identity matrix respectively. Since the closed loop system is stable, it follows that $\forall i \in \{1, \ldots, 2n\}$,

$$\mid \lambda_i(\bar{A}(w)) \mid = \mid \lambda_i(\bar{A}(w_0)) \mid \quad (3)$$

is in the open unit disk. It is shown in (3), if $C(z)$ is implemented by an infinite precision DCP, all different realization w achieve the same stability performance. However, in fact, $C(z)$ can only be implemented by a DCP with FWL. Due to the FWL effect, w is perturbed into $w + \Delta w$ and each element of Δw is bounded by $\varepsilon/2$, i.e.

$$\mu(\Delta w) = \max_{i\in\{1,\ldots,N\}} \mid \Delta w_i \mid \leq \varepsilon/2 \quad (4)$$

For a fixed point processor of B_s bits,

$$\varepsilon = 2^{-(B_s - B_X)} \quad (5)$$

where 2^{B_X} is the biggest normalization factor such that each parameter of $2^{-B_X}w$ is absolutely not bigger than 1. Here, for simply, (4) assume an equal upper bound for all elements. Actually, if an element is 1, 0 or -1, there is no FWL perturbation on this element. A less conservative assumption involving the trivial elements can be found in [4]. With the perturbation Δw, $\lambda_i(\bar{A}(w))$ is moved to $\lambda_i(\bar{A}(w+\Delta w))$ which may be outside the open unit disk. Thus, the closed-loop system designed stable may be unstable with FWL consideration.

Notice that FWL results different $\lambda_i(\bar{A}(w + \Delta w))$ for different realization w. we know that different realization w achieves different FWL stability performance. Next we will derive a FWL stability measure to describe the FWL stability performance: When Δw is small, $\forall i \in \{1, \ldots, 2n\}$, from a first order Taylor expansion, we have

$$\Delta\lambda_i = \lambda_i(\bar{A}(w+\Delta w)) - \lambda_i(\bar{A}(w)) \approx \sum_{j=1}^{N} \frac{\partial\lambda_i}{\partial w_j}\Delta w_j \quad (6)$$

and hence an upper bound of $\mid \Delta\lambda_i \mid$ can be derived:

$$\mid \Delta\lambda_i \mid \leq \sum_{j=1}^{N} \mid \frac{\partial\lambda_i}{\partial w_j} \mid\mid \Delta w_j \mid \leq \mu(\Delta w) \sum_{j=1}^{N} \mid \frac{\partial\lambda_i}{\partial w_j} \mid \quad (7)$$

Define

$$\mu_1(w) = \min_{i\in\{1,\ldots,2n\}} \frac{1 - \mid \lambda_i(\bar{A}(w)) \mid}{\sum_{j=1}^{N} \mid \frac{\partial\lambda_i}{\partial w_j} \mid} \quad (8)$$

When $\mu(\Delta w) < \mu_1(w)$, from (6)-(8), we have

$$| \lambda_i(\bar{A}(w+\Delta w)) | \leq | \lambda_i(\bar{A}(w)) | + | \Delta\lambda_i |$$
$$\leq | \lambda_i(\bar{A}(w)) | + \mu_1(w) \sum_{j=1}^{N} | \frac{\partial \lambda_i}{\partial w_j} | < 1 \quad (9)$$

which means that the closed-loop system remains stable under the FWL error Δw. In other words, for a given realization w, the closed-loop stability can tolerate those FWL perturbations Δw, whose elements have magnitudes less than $\mu_1(w)$. Hence $\mu_1(w)$ can be taked as an FWL stability measure: the larger $\mu_1(w)$ is, the larger FWL errors the closed-loop system can tolerate.

For computation of $\mu_1(w)$, the following theorem is important.

Theorem 1: Suppose $n \times n$ square matrix $\bar{A} = M_1 X M_2 + M_3 X M_4$ is diagonalizable and has $\{\lambda_i\}$ as its eigenvalues, matrix $X \in R^{p\times q}$, matrix M_1, M_2, M_3 and M_4 has proper dimension respectively. Let x_i be a right eigenvector of $\bar{A}$ corresponding to the eigenvalue λ_i. Denote $M_x = [x_1 \ x_2 \ \cdots \ x_n]$ and $M_y = [y_1 \ y_2 \ \cdots \ y_n] = M_x^{-H}$, where y_i is called the reciprocal left eigenvector corresponding to λ_i. Then

$$\frac{\partial \lambda_i}{\partial X} = \begin{bmatrix} \frac{\partial \lambda_i}{\partial x_{11}} & \cdots & \frac{\partial \lambda_i}{\partial x_{1q}} \\ \vdots & \cdots & \vdots \\ \frac{\partial \lambda_i}{\partial x_{p1}} & \cdots & \frac{\partial \lambda_i}{\partial x_{pq}} \end{bmatrix} = M_1^T y_i' x_i^T M_2^T + M_3^T y_i' x_i^T M_4^T \quad (10)$$

where superscript 'H' denotes the transpose and conjugate operation, superscript 'T' denotes the transpose operation. 'y_i'' is conjugate to y_i.

Proof: Let α be a variable indenpendent of M_1, M_2, M_3 and M_4. It follows from $y_i^H x_i = 1$ that

$$\frac{\partial y_i^H}{\partial \alpha} x_i + y_i^H \frac{\partial x_i}{\partial \alpha} = 0 \quad (11)$$

Notice that $\bar{A}x_i = \lambda_i x_i$ and $\lambda_i = y_i^H \bar{A} x_i$. Hence,

$$\frac{\partial \lambda_i}{\partial \alpha} = \frac{\partial y_i^H}{\partial \alpha} \bar{A} x_i + y_i^H \frac{\partial \bar{A}}{\partial \alpha} x_i + y_i^H \bar{A} \frac{\partial x_i}{\partial \alpha} \quad (12)$$

It follows from (11) and $y_i^H \bar{A} = \lambda_i y_i^H$ that

$$\begin{aligned} \frac{\partial \lambda_i}{\partial \alpha} &= \left(\frac{\partial y_i^H}{\partial \alpha} \lambda_i x_i + \lambda_i y_i^H \frac{\partial x_i}{\partial \alpha} \right) + y_i^H \frac{\partial \bar{A}}{\partial \alpha} x_i \\ &= y_i^H \frac{\partial \bar{A}}{\partial \alpha} x_i \\ &= y_i^H M_1 \frac{\partial X}{\partial \alpha} M_2 x_i + y_i^H M_3 \frac{\partial X}{\partial \alpha} M_4 x_i \quad (13) \end{aligned}$$

Let $\alpha = x_{kj}$. Then,

$$\frac{\partial \lambda_i}{\partial \alpha} = \left(y_i^H M_1\right)_k (M_2 x_i)_j + \left(y_i^H M_3\right)_k (M_4 x_i)_j \quad (14)$$

where $\left(y_i^H M_1\right)_k$, $(M_2 x_i)_j$, $\left(y_i^H M_3\right)_k$ and $(M_4 x_i)_j$ is the kth element of $y_i^H M_1$, the jth element of $M_2 x_i$, the kth element of $y_i^H M_3$ and the jth element of $M_4 x_i$ respectively. This leads to (10).

Using Theorem 1, we have

$$\begin{aligned} \frac{\partial \lambda_i}{\partial A} &= \begin{bmatrix} 0 & I_n \end{bmatrix} \begin{bmatrix} 0 & -J^T \\ 0 & I_n \end{bmatrix} y_i' x_i^T \begin{bmatrix} 0 & 0 \\ -K^T & I_n \end{bmatrix} \begin{bmatrix} 0 \\ I_n \end{bmatrix} \\ &= \begin{bmatrix} 0 & I_n \end{bmatrix} y_i' x_i^T \begin{bmatrix} 0 \\ I_n \end{bmatrix} \quad (15) \end{aligned}$$

$$\begin{aligned} \frac{\partial \lambda_i}{\partial B} &= \begin{bmatrix} 0 & I_n \end{bmatrix} \begin{bmatrix} 0 & -J^T \\ 0 & I_n \end{bmatrix} y_i' x_i^T \begin{bmatrix} 0 & 0 \\ -K^T & I_n \end{bmatrix} \begin{bmatrix} I_p \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & I_n \end{bmatrix} y_i' x_i^T \begin{bmatrix} 0 \\ -K^T \end{bmatrix} \quad (16) \end{aligned}$$

$$\begin{aligned} \frac{\partial \lambda_i}{\partial C} &= \begin{bmatrix} I_q & 0 \end{bmatrix} \begin{bmatrix} 0 & -J^T \\ 0 & I_n \end{bmatrix} y_i' x_i^T \begin{bmatrix} 0 & 0 \\ -K^T & I_n \end{bmatrix} \begin{bmatrix} 0 \\ I_n \end{bmatrix} \\ &= \begin{bmatrix} 0 & -J^T \end{bmatrix} y_i' x_i^T \begin{bmatrix} 0 \\ I_n \end{bmatrix} \quad (17) \end{aligned}$$

$$\frac{\partial \lambda_i}{\partial K} = \begin{bmatrix} -B_0^T & -B^T \end{bmatrix} y_i' x_i^T \begin{bmatrix} 0 \\ I_n \end{bmatrix} \quad (18)$$

$$\frac{\partial \lambda_i}{\partial J} = \begin{bmatrix} 0 & I_n \end{bmatrix} y_i' x_i^T \begin{bmatrix} C_0^T \\ -C^T \end{bmatrix} \quad (19)$$

With $\frac{\partial \lambda_i}{\partial A}$, $\frac{\partial \lambda_i}{\partial B}$, $\frac{\partial \lambda_i}{\partial C}$, $\frac{\partial \lambda_i}{\partial K}$, and $\frac{\partial \lambda_i}{\partial J}$, $\mu_1(w)$ can be computed easily using (8). Based on $\mu_1(w)$, we compute

$$\hat{B}_{s1}^{\min} = Int(-\log_2 \mu_1(w) - 1 + B_X) \quad (20)$$

where $Int(x)$ rounds x to the nearest integer towards $+\infty$. From the analysis of this section, we know the closed loop system is still stable when w is implemented by a DCP of at least $\hat{B}_{s1}^{\min}$ bits.

3 Optimal Realization

From the last section, we know that there are different realizations for a given $C(z)$, and the FWL stability measure $\mu_1(w)$ is a function of the realization w. Hence there is an interesting problem of finding out the realization such that $\mu_1(w)$ is maximized. This realization is called optimal realization in such a sense. The digital state-estimate feedback controller implemented with an optimal realization means the minimum hardware requirements in terms of less word length (i.e optimized controller data path hardware design) and such that the closed loop system remains stable.

Let x_{i0} be a right eigenvector of $\bar{A}(w_0)$ corresponding to the eigenvalue $\lambda_{i0} = \lambda_i(\bar{A}(w_0))$, y_{i0} be the reciprocal left eigenvector corresponding to x_{i0}. It is easy to see from (2) that $\forall i \in \{1, \cdots, 2n\}$, the right eigenvector of

$\bar{A}(w)$ corresponding to the eigenvalue $\lambda_i(\bar{A}(w)) = \lambda_{i0}$ is $x_i = \begin{bmatrix} I_n & 0 \\ 0 & T^{-1} \end{bmatrix} x_{i0} \in C^{2n}$, and the reciprocal left eigenvector is $y_i = \begin{bmatrix} I_n & 0 \\ 0 & T^T \end{bmatrix} y_{i0} \in C^{2n}$. Applying (15)–(19), we have

$$\frac{\partial \lambda_i}{\partial A} = [0 \quad T^T] y'_{i0} x^T_{i0} \begin{bmatrix} 0 \\ T^{-T} \end{bmatrix} \tag{21}$$

$$\frac{\partial \lambda_i}{\partial B} = [0 \quad T^T] y'_{i0} x^T_{i0} \begin{bmatrix} 0 \\ -K^T_0 \end{bmatrix} \tag{22}$$

$$\frac{\partial \lambda_i}{\partial C} = [0 \quad -J^T_0] y'_{i0} x^T_{i0} \begin{bmatrix} 0 \\ T^{-T} \end{bmatrix} \tag{23}$$

$$\frac{\partial \lambda_i}{\partial K} = [-B^T_0 \quad -B^T_0] y'_{i0} x^T_{i0} \begin{bmatrix} 0 \\ T^{-T} \end{bmatrix} \tag{24}$$

$$\frac{\partial \lambda_i}{\partial J} = [0 \quad T^T] y'_{i0} x^T_{i0} \begin{bmatrix} C^T_0 \\ -C^T_0 \end{bmatrix}. \tag{25}$$

For complex matrix $X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \cdots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \in C^{m\times n}$, denote

$$\|X\|_S = \sum_{i=1}^{m}\sum_{j=1}^{n} | x_{ij} | \tag{26}$$

We can describe the optimal FWL realization problem of state-estimate feedback controller as the optimization problem:

$$v = \frac{1}{\max_w \mu_1(w)} = \min_{\substack{T\in R^{n\times n} \\ \det(T)\neq 0}} \max_{i\in\{1,\cdots,2n\}} \frac{\sum_{j=1}^{N} | \frac{\partial \lambda_i}{\partial w_j} |}{1- | \lambda_{i0} |}$$
$$= \min_{\substack{T\in R^{n\times n} \\ \det(T)\neq 0}} \max_{i\in\{1,\cdots,2n\}} g(A,B,C,K,J,i) \tag{27}$$

where

$$g(A,B,C,K,J,i) = \frac{\|\frac{\partial \lambda_i}{\partial A}\|_S + \|\frac{\partial \lambda_i}{\partial B}\|_S + \|\frac{\partial \lambda_i}{\partial C}\|_S + \|\frac{\partial \lambda_i}{\partial K}\|_S + \|\frac{\partial \lambda_i}{\partial J}\|_S}{1- | \lambda_{i0} |} \tag{28}$$

From (21)–(25), we can define

$$f(T) = \max_{i\in\{1,\cdots,2n\}} g(A,B,C,K,J,i) \tag{29}$$

which is a function of T. Then the optimal state-estimate feedback controller realization problem can be posed as

$$v = \min_{\substack{T\in R^{n\times n} \\ \det(T)\neq 0}} f(T) \tag{30}$$

the above problem is a nonconvex nonlinear programming problem. We intend to search for the minimum of problem (30) with iterative optimization methods, i.e. a sequence $\{T_0, T_1, T_2, \cdots\}$ which converges to the minimum T_{opt} is generated. In the iterative procedure we can neglect the constraint $\det T \neq 0$, i.e. we solve the problem

$$v = \min_{T\in R^{n\times n}} f(T) \tag{31}$$

with iterative methods. There are two reasons for us to do so:

- $\Omega = \{T \mid \det T = 0, T \in R^{n\times n}\}$ is a very small set in space $R^{n\times n}$. Hence the case is rare that the iterate T_i moves into Ω when we search the space $R^{n\times n}$ for $T_{opt} \notin \Omega$ by an iterative sequence from the start point $T_0 \notin \Omega$.
- Even if it happens that T_i moves into Ω in the iterative procedure, we can add a small perturbation τI_n to T_i such that $T_i + \tau I_n \notin \Omega$. This small perturbation would not affect the convergence of the iterative sequence to T_{opt}.

In this paper, the simplex search method is applied to solve problem (31) which is an unconstrained nonconvex nonlinear programming problem. There are many existing optimization software which uses the simplex search method, for example, the *fmins* function in MATLAB Ver5.1 optimization toolbox. It is well known, even if for a nonconvex nonlinear programming problem, the simple search algorithm can always converge to a locally optimal point. In order to "globalize" the algorithm, we repeatedly run the algorithm starting from various initial point to obtain "randomized" solutions for v; then pick the smallest solution obtained.

4 Illustrative Example

To show how the optimization approach presented in this paper can be used efficiently for the parameterization issues of optimal FWL state-estimate feedback controller realization with improved stability bounds and minimum word-length requirements. We consider an example given in [6] to confirm our theoretical results.

The discrete time plant is given by

$$P(z) = \frac{0.0022(z+1)^2}{(z-0.9588)(z-0.9231)(z-0.8763)}$$

a state space description of $P(z)$ is

$$A_0 = \begin{bmatrix} 2.7582 & -2.5342 & 0.7756 \\ 1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$
$$C_0 = [0.0022 \quad 0.0044 \quad 0.0022]$$

Given the initial realization of the controller $C(z)$

$$A_{ini} = A_0, B_{ini} = B_0, C_{ini} = C_0,$$

$$K_{ini} = [0.4761 \quad -0.8183 \quad 0.3506],$$

$$J_{ini} = \begin{bmatrix} 118.2995 \\ 101.0891 \\ 81.8859 \end{bmatrix}$$

The corresponding transition matrix $\tilde{A}$ can then be formed using (2), from which the poles of the ideal closed loop system can be computed and given as

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{bmatrix} = \begin{bmatrix} 0.9067 \\ 0.8437 \\ 0.4532 \\ 0.7523 \\ 0.5761 \\ 0.6231 \end{bmatrix}$$

the corresponding eigenvectors x_{i0}, y_{i0} can be computed and hence problem (31) can be constructed. For problem (31), we get the solution using the simplex search method:

$$T_{opt} = \begin{bmatrix} 19.2056 & 1.9810 & -1.2562 \\ 8.6287 & 1.4078 & 8.9271 \\ -2.2033 & -1.0608 & 21.3406 \end{bmatrix}$$

and $v = 1.0525 \times 10^4$. The optimal realization corresponding to T_{opt} is

$$A_{opt} = \begin{bmatrix} 1.4973 & -0.0311 & -0.4183 \\ 0.7003 & 0.9125 & -0.5379 \\ 0.5937 & 0.1081 & 0.3484 \end{bmatrix},$$

$$B_{opt} = \begin{bmatrix} 0.1089 \\ -0.5619 \\ -0.0167 \end{bmatrix},$$

$$C_{opt} = [0.0754 \quad 0.0082 \quad 0.0835],$$

$$K_{opt} = [1.3101 \quad -0.5808 \quad -0.4223],$$

$$J_{opt} = \begin{bmatrix} 5.8678 \\ 5.8305 \\ 4.7327 \end{bmatrix}$$

The results for the initial realization and optimal realization are summarized in Table 1:

Table 1. Measures and stabilized word lengths

Realization	$\mu_1(w)$	$\hat{B}_{s1}^{min}$
w_{ini}	1.9810×10^{-5}	22
w_{opt}	9.5012×10^{-5}	16

The comparative results clearly show that optimal realization w_{opt} needs only 16 bits (including fractional part and part of integer) and provides larger stability measure while the initial realization w_{ini} requires 22 bits (including fractional part and part of integer) with lower stability bound.

5 Conclusions

In this paper we have presented an efficient approach for the stability measure of state estimate feedback controllers with FWL consideration. It has also been shown that the optimal realization problem for state estimate feedback controller with FWL consideration can be interpreted as a nonlinear programming problem. The computation of the relevant FWL optimization problem was solved using the simplex search algorithm to illustrate that such problem can be efficiently and easily computed using existing mathematical programming techniques. The theoretical results were verified using a numerical example which illustrate that the optimum realization based on the optimization method presented here greatly improves the stability robustness of the relevant controller realizations with minimum word-length characteristics compared to non-optimal realizations.

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