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Nonlinear Beamforming for Multiple-Antenna Assisted QPSK Wireless Systems

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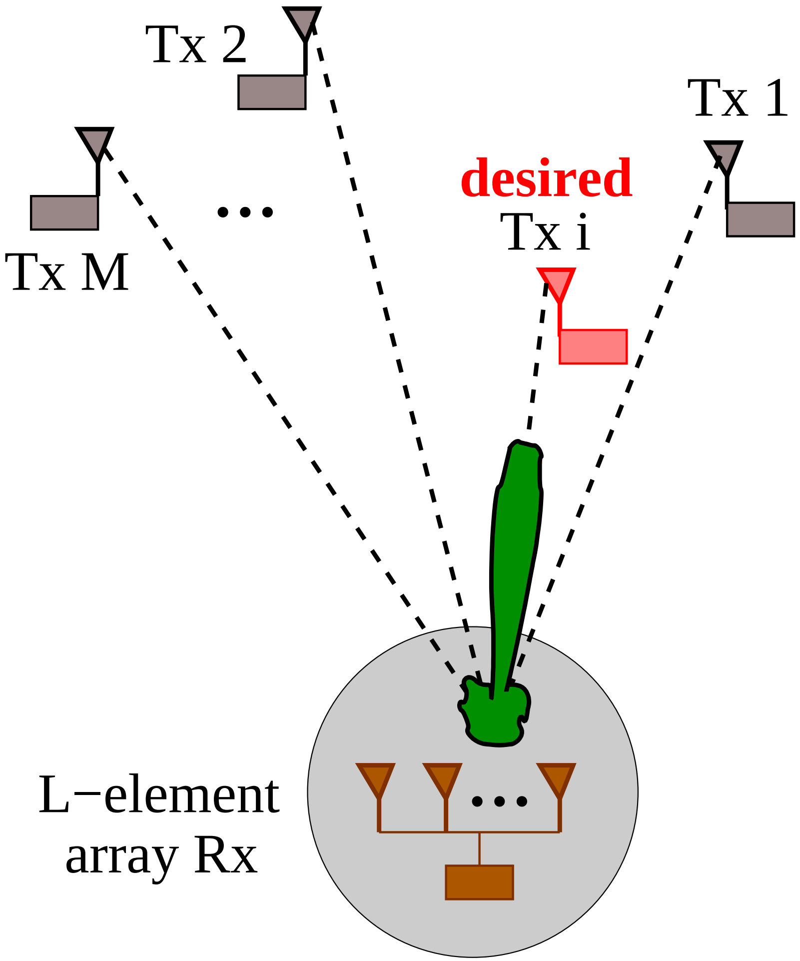
Outline

- Existing linear beamforming techniques, and motivations for nonlinear beamforming or detection
- Signal model and optimal Bayesian detection with an inherent symmetry property for QPSK beamforming
- Symmetric radial basis function network for nonlinear beamforming and adaptive clustering algorithm
- Simulation investigation, and performance comparison

Motivations



- Classical beamforming is **linear** with a **beampattern** interpretation of beamformer's weight vector:
  - maximise response at desired user **direction** and place nulls at interferers' directions, **must** $L \geq M$
  - similar to **zero-forcing** equalisation, and suffers from **noise enhancement**
- Standard linear beamforming is **minimum mean square error** (L-MMSE):
  - better balances nullifying interferers and enhancing noise

Motivations (continue)

- State-of-the-art for linear beamforming is **minimum bit error rate** (L-MBER) technique, and in comparsion with L-MMSE it offers
  - Better system BER performance, and larger user capacity
- Beamforming can be viewed as **classification**, which classifies received channel-impaired signal into most-likely transmitted symbol point
- In comparison with linear beamforming, **nonlinear** detection offers
  - significantly better BER performance and much larger user capacity, at cost of higher complexity
- With **posterior** or **conditional probabilities** as **generalised beam-pattern** interpretation
  - This nonlinear detection can be viewed as **nonlinear beamforming**

Signal Model

- M single-transmit-antenna users transmit on same carrier, receiver is equipped with L-element **antenna array**, channels are non-dispersive
- Received signal vector $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_L(k)]^T$ is

$$\mathbf{x}(k) = \mathbf{P}\,\mathbf{b}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k)$$

- $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots n_L(k)]^T$ is noise vector, and **system matrix**

$$\mathbf{P} = [A_1\mathbf{s}_1\ A_2\mathbf{s}_2 \cdots A_M\mathbf{s}_M]$$

- $\mathbf{s}_i$ is **steering vector** of source i, A_i is i-th non-dispersive channel tap
- User i is **desired** user, and transmitted symbol vector $\mathbf{b}(k) = [b_1(k)\ b_2(k) \cdots b_M(k)]^T$ with QPSK symbol set

$$b_m(k) \in \{b^{[1]} = +1+j,\ b^{[2]} = -1+j, b^{[3]} = -1-j,\ b^{[4]} = +1-j\}, 1 \le m \le M$$

Signal Space

- Denote $N_b = 4^M$ **legitimate sequences** of $\mathbf{b}(k)$ as $\mathbf{b}_q$, $1 \le q \le N_b$
- Noiseless **channel state** $\bar{\mathbf{x}}(k)$ takes values from set

$$\bar{\mathbf{x}}(k) \in \mathcal{X} = \{\bar{\mathbf{x}}_q = \mathbf{P}\,\mathbf{b}_q, 1 \le q \le N_b\}$$

which can be divided into **four subsets** conditioned on $b_i(k) = b^{[m]}$

$$\mathcal{X}^{[m]} \triangleq \{\bar{\mathbf{x}}_q^{[m]} \in \mathcal{X}, 1 \le q \le N_{sb} : b_i(k) = b^{[m]}\}, \ 1 \le m \le 4$$

- **Conditional probabilities** of receiving $\mathbf{x}(k)$ given $b_i(k) = b^{[m]}$ are

$$p^{[m]}(\mathbf{x}(k)) = \sum_{q=1}^{N_{sb}} \beta_q e^{-\frac{\|\mathbf{x}(k) - \bar{\mathbf{x}}_q^{[m]}\|^2}{2\sigma_n^2}}, \ 1 \le m \le 4$$

$N_{sb} = N_b/4 = 4^{M-1}$, noise power is $2\sigma_n^2$ and all priors β_q are equal

- $p^{[m]}(\mathbf{x}(k))$ can be interpreted as **generalised beampatterns**

Optimal Bayesian Detector

- **Optimal detection** strategy is

$$\hat{b}_i(k) = b^{[m^*]} \quad \text{with} \quad m^* = \arg \max_{1 \le m \le 4} p^{[m]}(\mathbf{x}(k))$$

- Define complex-valued Bayesian **decision variable**

$$y_{\text{Bay}}(k) \triangleq b^{[1]} \cdot p^{[1]}(\mathbf{x}(k)) + b^{[2]} \cdot p^{[2]}(\mathbf{x}(k)) + b^{[3]} \cdot p^{[3]}(\mathbf{x}(k)) + b^{[4]} \cdot p^{[4]}(\mathbf{x}(k))$$

- Optimal **Bayesian** detection is: $\hat{b}_i(k) = \text{sgn}(y_{\text{Bay}}(k))$, where

$$\text{sgn}(y) = \begin{cases} b^{[1]} = +1 + j, & y_R \ge 0 \text{ and } y_I \ge 0, \\ b^{[2]} = -1 + j, & y_R < 0 \text{ and } y_I \ge 0, \\ b^{[3]} = -1 - j, & y_R < 0 \text{ and } y_I < 0, \\ b^{[4]} = +1 - j, & y_R \ge 0 \text{ and } y_I < 0, \end{cases}$$

Symmetry of Bayesian Solution

- Four state subsets satisfy following **symmetric** properties

$$\mathcal{X}^{[2]} = +j \cdot \mathcal{X}^{[1]}, \; \mathcal{X}^{[3]} = -1 \cdot \mathcal{X}^{[1]}, \; \mathcal{X}^{[4]} = -j \cdot \mathcal{X}^{[1]}$$

- Thus **Bayesian solution** becomes, for $\bar{\mathbf{x}}_q^{[1]} \in \mathcal{X}^{[1]}$,

$$y_{\text{Bay}}(k) = \sum_{q=1}^{N_{sb}} \Bigg\{ b^{[1]}\beta \cdot e^{-\frac{\|\mathbf{x}(k)-\bar{\mathbf{x}}_q^{[1]}\|^2}{2\sigma_n^2}} + b^{[2]}\beta \cdot e^{-\frac{\|\mathbf{x}(k)-j\cdot\bar{\mathbf{x}}_q^{[1]}\|^2}{2\sigma_n^2}}$$

$$+ b^{[3]}\beta \cdot e^{-\frac{\|\mathbf{x}(k)+\bar{\mathbf{x}}_q^{[1]}\|^2}{2\sigma_n^2}} + b^{[4]}\beta \cdot e^{-\frac{\|\mathbf{x}(k)+j\cdot\bar{\mathbf{x}}_q^{[1]}\|^2}{2\sigma_n^2}} \Bigg\}$$

- If system **channel matrix** $\mathbf{P}$ can be estimated, as in **uplink**, subset $\mathcal{X}^{[1]}$ can be calculated and Bayesian solution is specified
- In **downlink**, receiver only has access to desired user's training data, estimating $\mathbf{P}$ is difficult, and other adaptive means has to be adopted

Symmetric RBF Network

- Consider generic **radial basis function** network

$$y_{\text{RBF}}(k) = \sum_{q=1}^{N_c} \Big\{ \alpha_q^{[1]} \varphi(\mathbf{x}(k); \mathbf{c}_q, \sigma_q^2) + \alpha_q^{[2]} \varphi(\mathbf{x}(k); j\mathbf{c}_q, \sigma_q^2)$$

$$+ \alpha_q^{[3]} \varphi(\mathbf{x}(k); -\mathbf{c}_q, \sigma_q^2) + \alpha_q^{[4]} \varphi(\mathbf{x}(k); -j\mathbf{c}_q, \sigma_q^2) \Big\}$$

- N_c is number of RBF units, $\varphi(\bullet)$ is usual RBF function, $\mathbf{c}_q$ RBF **centres**, and σ_q^2 RBF variances
- This RBF network has the same **symmetric property** as the Bayesian detector $y_{\text{Bay}}(k)$
- $N_c = N_{sb}$, all $\alpha_q^{[m]} = \beta b^{[m]}$, all $\sigma_q^2 = \hat{\sigma}_n^2$: N_{sb} is usually known, β is any positive number, and $\hat{\sigma}_n^2$ an estimate of noise variance
- One only needs to determine all **centres** $\mathbf{c}_q$

Clustering

- Given **training data** $\{\mathbf{x}(k), b_i(k)\}$, enhanced κ-means **clustering**:

$$\mathbf{c}_l(k) = \mathbf{c}_l(k-1) + \mu_c \mathcal{M}_l(\check{\mathbf{x}}(k))(\check{\mathbf{x}}(k) - \mathbf{c}_l(k-1))$$

- μ_c is step size,

$$\check{\mathbf{x}}(k) = \begin{cases} +1 \cdot \mathbf{x}(k), & b_i(k) = b^{[1]}, \\ -j \cdot \mathbf{x}(k), & b_i(k) = b^{[2]}, \\ -1 \cdot \mathbf{x}(k), & b_i(k) = b^{[3]}, \\ +j \cdot \mathbf{x}(k), & b_i(k) = b^{[4]}, \end{cases}$$

- **membership function**

$$\mathcal{M}_l(\mathbf{x}) = \begin{cases} 1, & \text{if } \bar{v}_l \|\mathbf{x} - \mathbf{c}_l\|^2 \le \bar{v}_q \|\mathbf{x} - \mathbf{c}_q\|^2, \forall q \neq l, \\ 0, & \text{otherwise,} \end{cases}$$

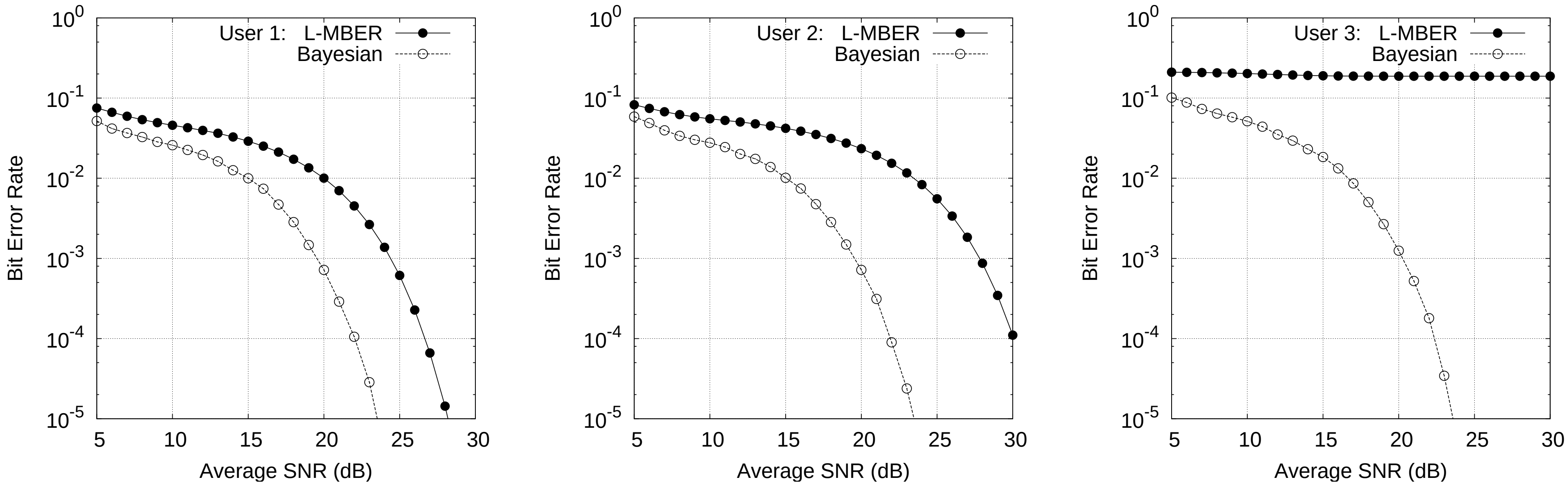
- cluster **variation**

$$\bar{v}_l(k) = \mu_v \bar{v}_l(k-1) + (1-\mu_v)\mathcal{M}_l(\check{\mathbf{x}}(k)) \|\check{\mathbf{x}}(k) - \mathbf{c}_l(k-1)\|^2$$

μ_v slightly less than 1.0, all $\bar{v}_l(0)$ set to same small number

Simulation Example

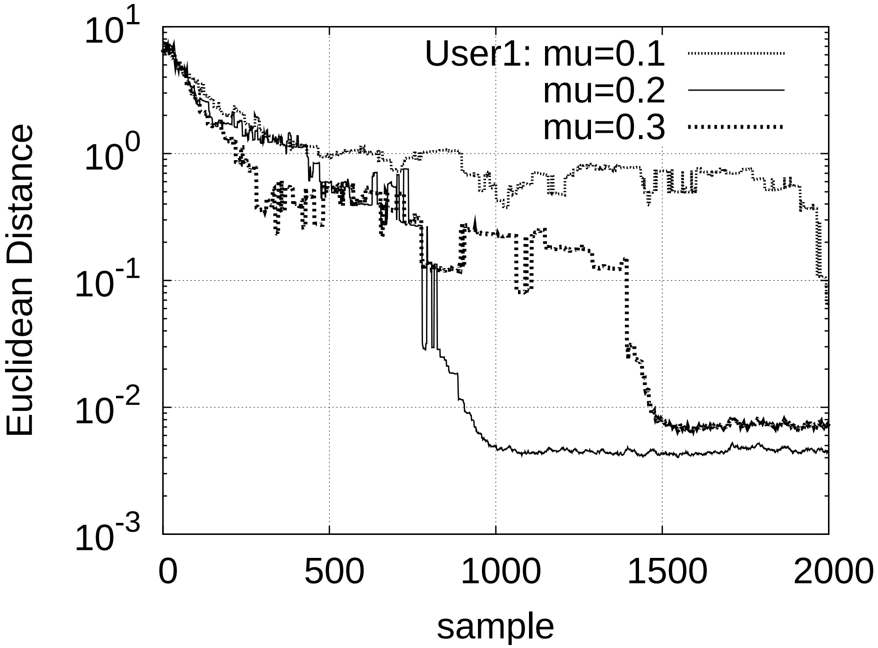
- 2-element array with half wavelength spacing, three equal-power QPSK users with **angles of arrival** $\rightarrow$ user 1: 15°, user 2: -60°, user 3: 45°
- **Bit error rate** comparison of theoretical **linear** MBER beamforming and **nonlinear** Bayesian beamforming



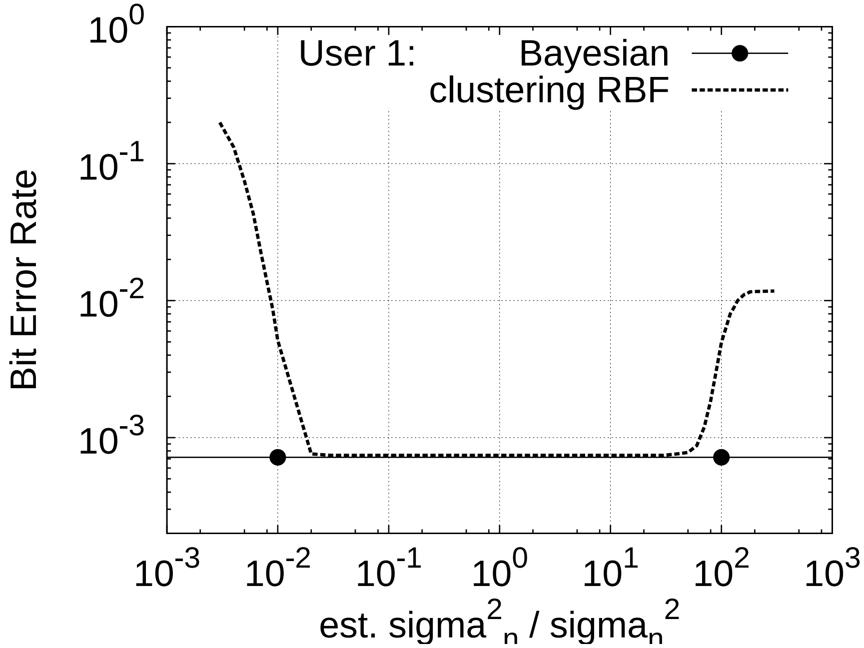
Clustering RBF Beamforming

- User-one, SNR= 20 dB, average over 10runs
- (a) **convergence** of clustering, Euclidean **distance** between RBF centres and true channel states, and (b) **insensitivity** to RBF variance

(a)

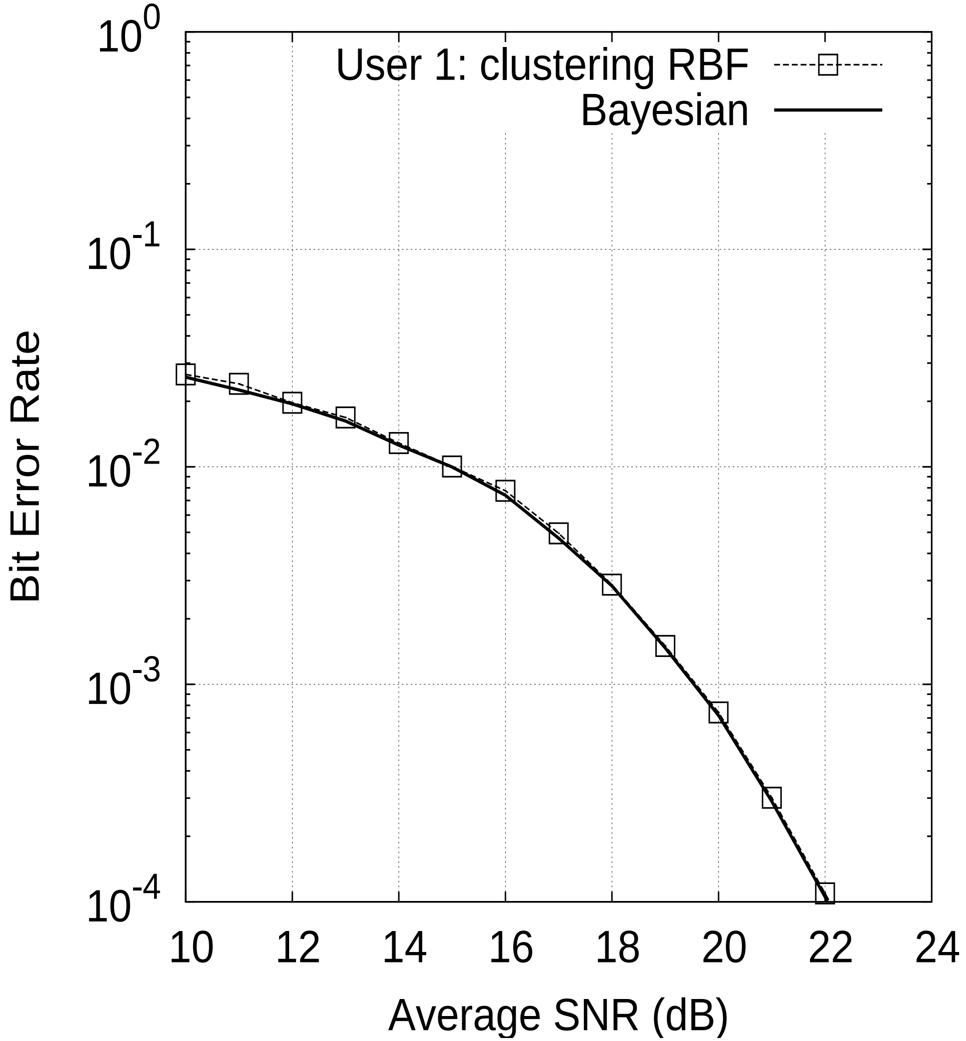


(b)



RBF Beamforming Performance

- **Bit error rate** of **clustering** RBF beamforming for user one, in comparison with optimal **Bayesian** beamforming based on perfect channel knowledge



Conclusions

- Nonlinear beamforming achieves significantly smaller system bit error rate and larger user capacity
- Optimal Bayesian beamforming solution for QPSK has an inherent symmetry structure
- A novel symmetric radial basis function network has been proposed for QPSK nonlinear beamforming
- An adaptive algorithm for downlink senario: cluster-variation enhanced clustering

THANK YOU.

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