Multiple Hyperplane Detector for Implementing the Asymptotic Bayesian Decision Feedback Equalizer

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Motivations

ISI corrupted multi-level M-PAM symbols:

- Bayesian or MAP symbol-decision DFE is optimal, but high complexity

  $\star$ Hypersurface separating two neighbouring signal classes

- Conventional *linear-combiner* DFE is very simple

  $\star$ Hyperplane separating two neighbouring signal classes

- Can we have multiple linear discriminant detector, which achieves Bayesian DFE performance at least for large channel SNR?

  $\Rightarrow$ high performance with simplicity

  $\star$ Several hyperplanes separating two neighbouring signal classes?

Previous Works

For the binary 2-PAM case:

- Signal space partitioning using multiple hyperplanes (Kim & Moon, ICC'98; Trans COM 2000)
- Multiple hyperplane detector design realizing asymptotic Bayesian DFE (Chen *et al*, Trans SP 2000)

This work extends it to the general M-PAM case:

$$y(k) = \sum_{i=0}^{n_a-1} a_i s(k-i) + e(k)$$

$$s(k) \in \{s_i = 2i - M - 1, \ 1 \leq i \leq M\}$$

$$\mathbf{y}(k) = [y(k) \cdots y(k-m+1)]^T \ \& \ \hat{\mathbf{s}}_b(k) = [\hat{s}(k-d-1) \cdots \hat{s}(k-d-n)]^T$$

$$\Rightarrow \hat{s}(k-d) \ \text{ of } \ s(k-d)$$

Choose $d = n_a - 1$, $m = n_a$ and $n = n_a - 1$

$$\mathbf{y}(k) = \mathbf{F}_1 \ \mathbf{s}_f(k) + \mathbf{F}_2 \ \mathbf{s}_b(k) + \mathbf{e}(k)$$

Assume $\mathbf{s}_b(k) = \hat{\mathbf{s}}_b(k)$, “space translation”:

$$\mathbf{r}(k) \triangleq \mathbf{y}(k) - \mathbf{F}_2 \ \hat{\mathbf{s}}_b(k)$$

As $\mathbf{s}_f(k) \in \{\mathbf{s}_{fj}, \ 1 \le j \le N_f = M^{d+1}\}$, channel state set:

$$R \triangleq \{\mathbf{r}_j = \mathbf{F}_1 \ \mathbf{s}_{fj}, \ 1 \le j \le N_f\}$$

M conditional subsets:

$$R^{(i)} \triangleq \{\mathbf{r}_j \in R : \ s(k-d) = s_i\}, \ 1 \le i \le M$$

Optimal Bayesian Detector

M decision variables

$$\rho_i(\mathbf{r}(k)) = \sum_{\mathbf{r}_j \in R^{(i)}} e^{-\frac{\left\|\mathbf{r}(k)-\mathbf{r}_j\right\|^2}{2\sigma_e^2}}, \; 1 \le i \le M$$

Minimum-error-rate decision

$$\hat{s}(k-d) = s_{i^*} \;\; \text{with} \;\; i^* = \arg \max_{1 \le i \le M} \{\rho_i(\mathbf{r}(k))\}$$

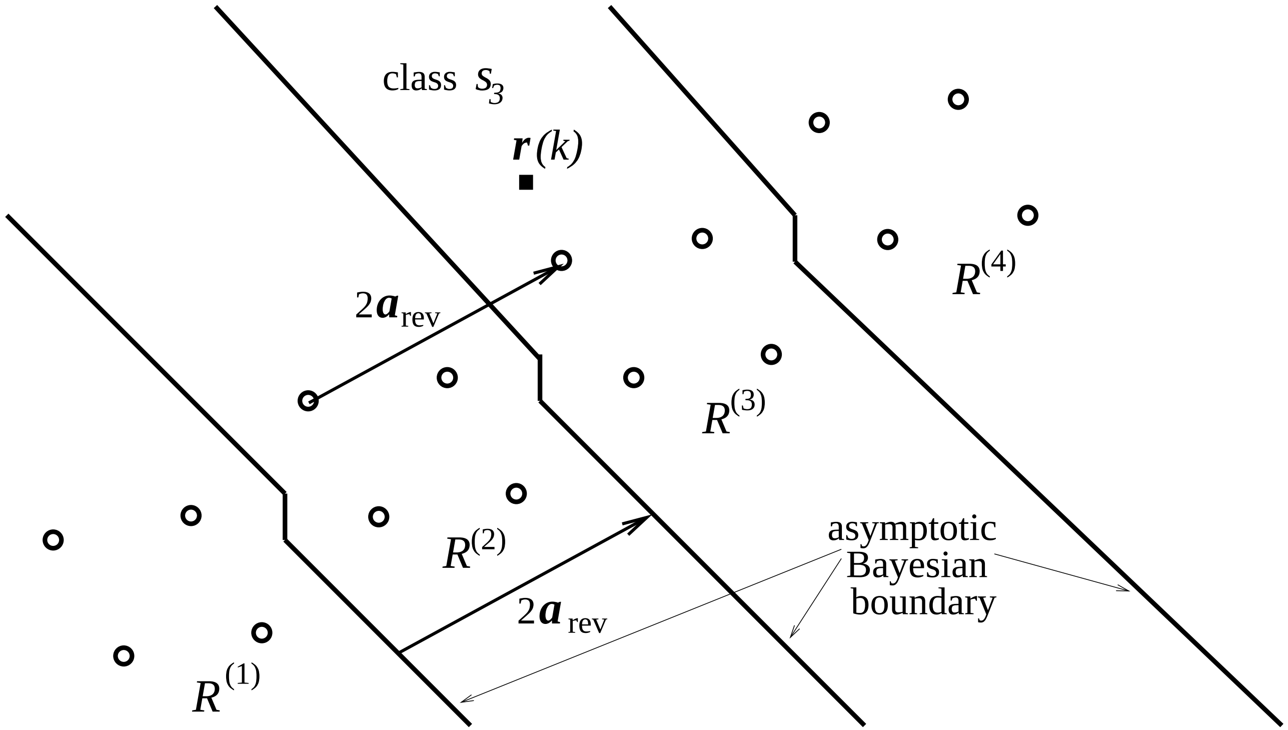
Lemma 1. For $1 \le i \le M-1$,

$$R^{(i+1)} = R^{(i)} + 2\mathbf{a}_{\text{rev}}$$

$R^{(i)}$ and $R^{(i+1)}$ are linearly separable, where $\mathbf{a}_{\text{rev}} = [a_{n_a-1} \cdots a_1 \; a_0]^T$.

Lemma 2. Asymptotically ($\sigma_e^2 \to 0$), optimal decision boundary separating two neighbouring signal classes is piecewise linear and made up of a set of L hyperplanes. Each of these hyperplanes is defined by a pair of *dominant* states.

⋆ Only need to consider one boundary or two neighbouring classes. ⋆



Multiple-Hyperplane Detector

Definition 1. A pair $(\mathbf{r}^{(+)} \in R^{(i+1)}, \mathbf{r}^{(-)} \in R^{(i)})$ is said to be *dominant* if $\forall \mathbf{r}_j \in R^{(i)} \bigcup R^{(i+1)}$, $\mathbf{r}_j \neq \mathbf{r}^{(+)}$ and $\mathbf{r}_j \neq \mathbf{r}^{(-)}$:

$$\|\mathbf{r}_j - \mathbf{r}_0\|^2 > \|\mathbf{r}^{(+)} - \mathbf{r}_0\|^2 \quad \text{where} \quad \mathbf{r}_0 = \frac{\mathbf{r}^{(+)} + \mathbf{r}^{(-)}}{2}$$

• Set of dominant state pairs $\{\mathbf{r}_l^{(+)}, \mathbf{r}_l^{(-)}\}_{l=1}^{L}$ easily found. Each pair defines a *canonical* hyperplane that is a part of the asymptotic optimal boundary

$$H_l(\mathbf{r}) = \mathbf{w}_l^T \mathbf{r} + b_l = 0$$

$$\mathbf{w}_l = \frac{2\left(\mathbf{r}_l^{(+)} - \mathbf{r}_l^{(-)}\right)}{\|\mathbf{r}_l^{(+)} - \mathbf{r}_l^{(-)}\|^2} \qquad b_l = -\frac{(\mathbf{r}_l^{(+)} - \mathbf{r}_l^{(-)})^T(\mathbf{r}_l^{(+)} + \mathbf{r}_l^{(-)})}{\|\mathbf{r}_l^{(+)} - \mathbf{r}_l^{(-)}\|^2}$$

Definition 2. $\mathbf{r}_j \in R^{(i)} \bigcup R^{(i+1)}$ is said to be *sufficiently separable* by H_l if H_l can separate $\mathbf{r}_j$ correctly with $|\mathbf{w}_l^T \mathbf{r}_j + b_l| \geq 1$.

- Test separability for all $\mathbf{r}_j \in R^{(i)} \bigcup R^{(i+1)}$ to generate separability table

- Construct convex region $\mathcal{R}_q^{(+,i)}$ covering each $\mathbf{r}_q^{(+)} \in R^{(i+1)}$ by intersecting separable hyperplanes $\rightarrow$ logic AND

- Construct decision region $\mathcal{R}^{(+,i)}$ by the union of all $\mathcal{R}_q^{(+,i)}$ $\rightarrow$ logic OR

$\star$ Detector: where $\mathbf{r}(k)$ is in relation to $\mathcal{R}^{(+,i)}$, $1 \leq i \leq M-1$ $\star$

	Full Bayesian	Multiple-hyperplane
Additions	$2n_a M^{n_a} - M$	$(n_a + M - 2)L$
Multiplications	$(n_a + 1)M^{n_a}$	$n_a L$
e^x	M^{n_a}	—

Channel $0.4 + 1.0z^{-1} + 0.6z^{-2}$, 4-PAM

5 pairs dominant states found for $R^{(2)}$ and $R^{(3)}$ $\rightarrow$ 5 separating hyperplanes

Separability Table

	$R^{(2)}$															
H_1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
H_2	0	0	0	1	0	0	1	1	1	1	1	1	1	1	1	1
H_3	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
H_4	0	0	0	0	0	0	0	1	0	0	1	1	1	1	1	1
H_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	$R^{(3)}$															
H_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
H_2	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0
H_3	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
H_4	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0
H_5	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

$$\mathcal{R}^{(+,2)} = \left(\mathcal{H}_1 \bigcap \mathcal{H}_2\right) \bigcup \left(\mathcal{H}_3 \bigcap \mathcal{H}_4\right) \bigcup \mathcal{H}_5$$

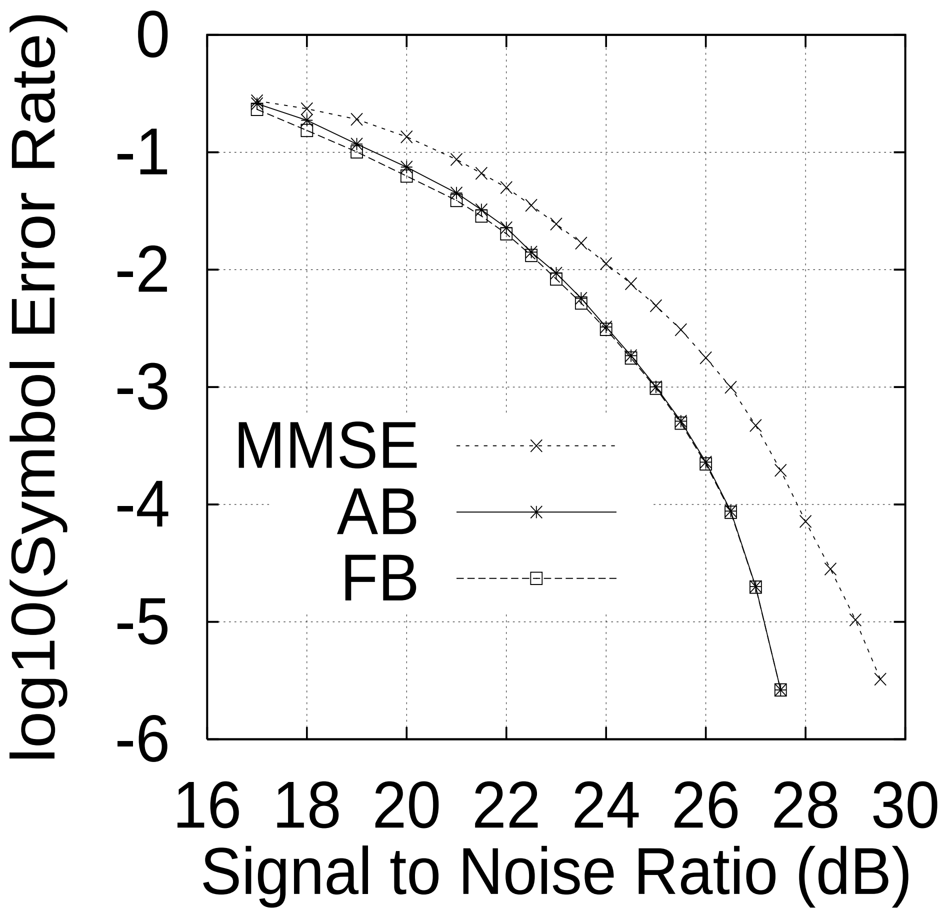
where $\mathcal{H}_l = \{\mathbf{r} : H_l(\mathbf{r}) \geq 0\}$. $\mathcal{R}^{(+,1)}$, $\mathcal{R}^{(+,3)}$ by shifting $\mathcal{R}^{(+,2)}$ accordingly

Channel $0.4 + 1.0z^{-1} + 0.6z^{-2}$, 4-PAM

Detected symbol feedback

	Full Baye	Multi hyp
adds	380	25
muls	256	15
exp	64	–

MMSE: linear MMSE DFE
AB: 5-hyperplane detector
FB: Full Bayesian



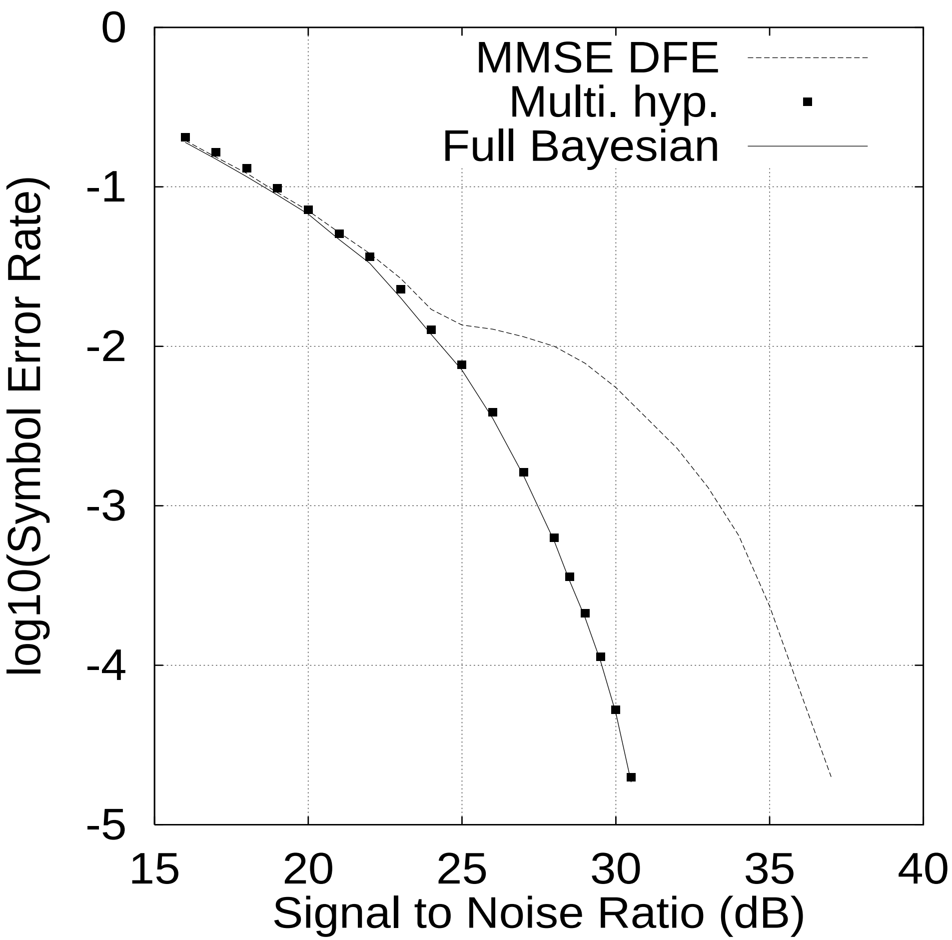
Channel $0.3 + 1.0z^{-1} - 0.3z^{-2}$, 8-PAM

19 hyperplanes

$$\mathcal{R}^{(+,4)} = (\mathcal{H}_1 \bigcap \mathcal{H}_5) \bigcup (\mathcal{H}_2 \bigcap \mathcal{H}_3 \bigcap \mathcal{H}_5) \bigcup (\mathcal{H}_4 \bigcap \mathcal{H}_5) \bigcup (\mathcal{H}_6 \bigcap \mathcal{H}_7) \bigcup (\mathcal{H}_8 \bigcap \mathcal{H}_9) \bigcup (\mathcal{H}_{10} \bigcap \mathcal{H}_{11}) \bigcup (\mathcal{H}_{12} \bigcap \mathcal{H}_{13}) \bigcup (\mathcal{H}_{14} \bigcap \mathcal{H}_{15}) \bigcup (\mathcal{H}_{16} \bigcap \mathcal{H}_{17}) \bigcup (\mathcal{H}_{18} \bigcap \mathcal{H}_{19})$$

Detected symbol feedback

	Full Baye	Multi hyp
adds	3064	171
muls	2048	57
exp	512	–



Conclusions

- Multiple-hpyerplane detector design for M-PAM case
  - ⋆ Design process simple and straightforward
  - ⋆ Asymptotically, realizes optimal Bayesian performance
    finite SNR, closely approximates Bayesian performance
  - ⋆ Complexity reduction particularly significant for high-order M
- For non-adaptive implementation, no need to use full Bayesian DFE