Concurrent Constant Modulus Algorithm and Soft Decision Directed Scheme for Fractionally-Spaced Blind Equalization

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Overview

Constant modulus algorithm aided soft decision directed scheme for low complexity blind equalization of high-order QAM channels

- Existing works:

  Constant modulus algorithm, concurrent CMA and decision directed scheme (De Castro *et al*, ICC2001)

- Concurrent CMA and soft decision directed scheme:

  Simpler operational requirements, faster convergence and similar steady-state equalization performance, compared with CMA+DD scheme

- Comparative simulation results

Constant Modulus Algorithm

CMA is a popular choice for blind equalization of high-order QAM channels

⇑ Very simple, need to know little e.g. robust to carrier recovery error, capable of opening initially closed eye

⇓ Steady-state performance may not be sufficiently accurate to achieve adequate BER

◯ A solution is to switch to (hard) decision directed adaptation after convergence

◯ When to switch ? and can it be switched to ?

CMA and DD

De Castro *et al* (2001) split equalizer into a CMA sub-equalizer and a DD sub-equalizer: $\mathbf{w} = \mathbf{w}_c + \mathbf{w}_d$

1. CMA adaptation: with $y(k) = \mathbf{w}_c^T(k)\mathbf{r}(k) + \mathbf{w}_d^T(k)\mathbf{r}(k)$

$$\left.\begin{aligned} \epsilon(k) &= y(k)(\Delta_2 - |y(k)|^2) \\ \mathbf{w}_c(k+1) &= \mathbf{w}_c(k) + \mu_c \epsilon(k)\mathbf{r}^*(k) \end{aligned}\right\}$$

2. DD adaptation: with $\tilde{y}(k) = \mathbf{w}_c^T(k+1)\mathbf{r}(k) + \mathbf{w}_d^T(k)\mathbf{r}(k)$

$$\mathbf{w}_d(k+1) = \mathbf{w}_d(k) + \mu_d \delta(\mathcal{Q}[\tilde{y}(k)] - \mathcal{Q}[y(k)])(\mathcal{Q}[y(k)] - y(k))\mathbf{r}^*(k)$$

   where $\mathcal{Q}[\bullet]$ denotes quantization operation or hard decision, $\delta(x) = 1$ if $x = 0 + j0$ and $\delta(x) = 0$ if $x \neq 0 + j0$

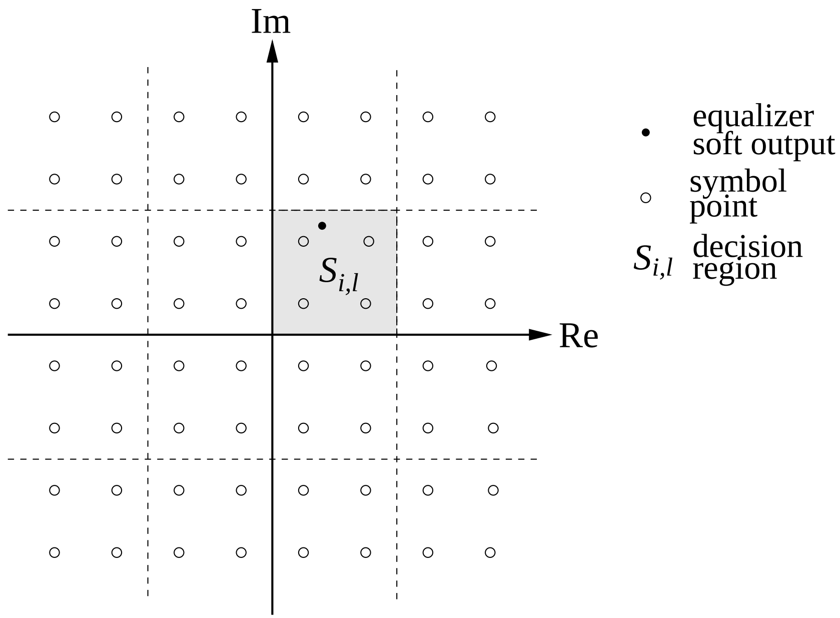
○ $\mathbf{w}_d$ is updated only if hard decisions before and after CMA adaptation are the same, to reduce error propagation due to incorrect decisions

Motivation for Soft DD

◯ After equalization is accomplished, the *a posteriori* PDF of $y(k)$ is approximately:

$$p(\mathbf{w}, y(k)) \approx \sum_{q=1}^{Q} \sum_{l=1}^{Q} \frac{p_{ql}}{2\pi\rho} \exp\left(- \frac{|y(k) - s_{ql}|^2}{2\rho} \right)$$

where s_{ql} are constellation points of Q^2-QAM, p_{ql} are the *a priori* probabilities of s_{ql}, and ρ is variance of $y(k)$



◯ Divide complex plane into $Q^2/4$ regular regions, each region $S_{i,l}$ contains four symbol points

If $y(k)$ is within $S_{i,l}$, a local approximation to the *a posteriori* PDF of $y(k)$ is

$$\hat{p}(\mathbf{w}, y(k)) \approx \sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} \frac{1}{8\pi\rho} \exp\left(- \frac{|y(k) - s_{pq}|^2}{2\rho} \right)$$

Motivation (continue)

◯ SDD equalizer is designed to maximize log of the local *a posteriori* PDF

$$\bar{J}_{\mathrm{LMAP}}(\mathbf{w}) = \mathsf{E}[J_{\mathrm{LMAP}}(\mathbf{w}, y(k))]$$

by adjusting $\mathbf{w}_d$ using a stochastic gradient algorithm, where

$$J_{\mathrm{LMAP}}(\mathbf{w}, y(k)) = \rho \log\left(\hat{p}(\mathbf{w}, y(k))\right)$$

◯ Stochastic gradient of $J_{\mathrm{LMAP}}(\mathbf{w}, y(k))$ is

$$\frac{\partial J_{\mathrm{LMAP}}(\mathbf{w}, y(k))}{\partial \mathbf{w}_d} = \frac{\sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} \exp\left(-\frac{|y(k)-s_{pq}|^2}{2\rho}\right)(s_{pq} - y(k))}{\sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} \exp\left(-\frac{|y(k)-s_{pq}|^2}{2\rho}\right)} \mathbf{r}^*(k)$$

◯ ρ is typically less than half the distance between neighbouring symbol points

◯ Rather than committed to a single hard decision $\mathcal{Q}[y(k)]$, tentative decisions are considered in a local region $S_{i,l}$ that includes $\mathcal{Q}[y(k)]$

CMA and SDD

◯ Operations of CMA and SDD sub-equalizers are truly concurrent:

- With $y(k) = \mathbf{w}_c^T(k)\mathbf{r}(k) + \mathbf{w}_d^T(k)\mathbf{r}(k)$

$$\text{CMA:} \quad \left.\begin{aligned} \epsilon(k) &= y(k)(\Delta_2 - |y(k)|^2) \\ \mathbf{w}_c(k+1) &= \mathbf{w}_c(k) + \mu_c \epsilon(k)\mathbf{r}^*(k) \end{aligned}\right\}$$

$$\text{SDD:} \quad \mathbf{w}_d(k+1) = \mathbf{w}_d(k) + \mu_d \frac{\partial J_{\text{LMAP}}(\mathbf{w}(k), y(k))}{\partial \mathbf{w}_d}$$

◯ Computational complexity is less than CMA+DD:

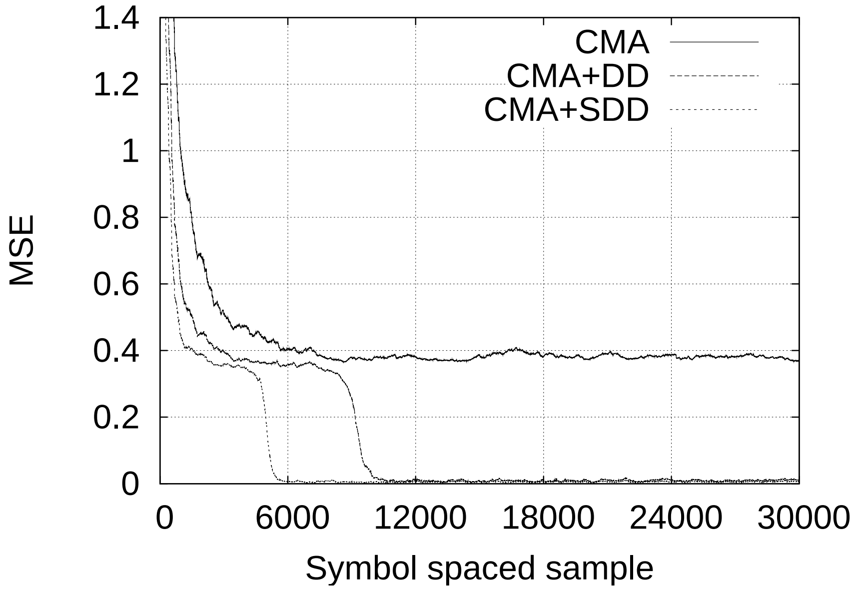
equalizer	multiplications	additions	$\exp(\bullet)$
CMA	$8 \times m_L + 6$	$8 \times m_L$	–
CMA+DD	$16 \times m_L + 8$	$20 \times m_L$	–
CMA+SDD	$12 \times m_L + 29$	$14 \times m_L + 21$	4

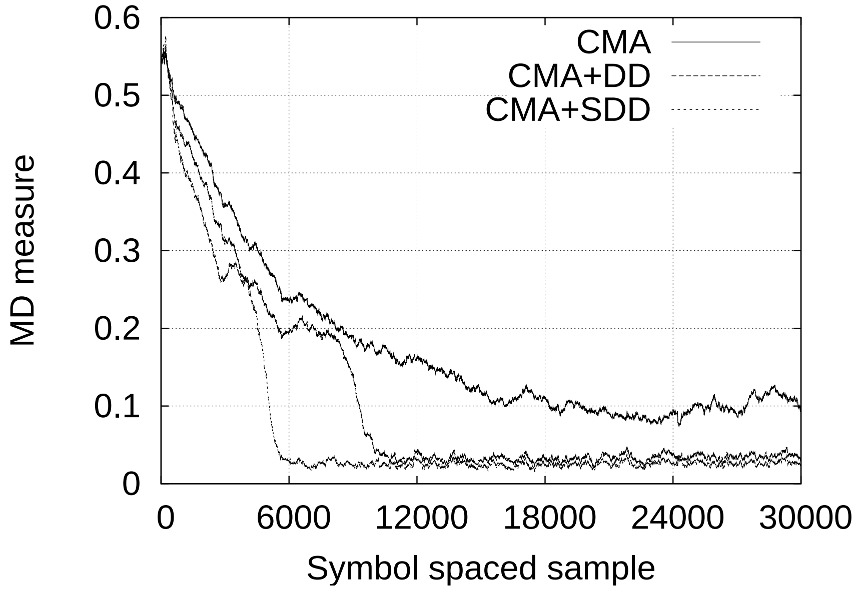
where m_L is equalizer length

Simulation (Fixed SISO Channel)

- $T_s/2$-spaced 22-tap channel and 26 tap equalizer, where T_s is symbol period, 256-QAM
- Let $\{f_i\}_{i=0}^{N_f-1}$ be combined impulse response of channel and equalizer. Maximum distortion is defined by

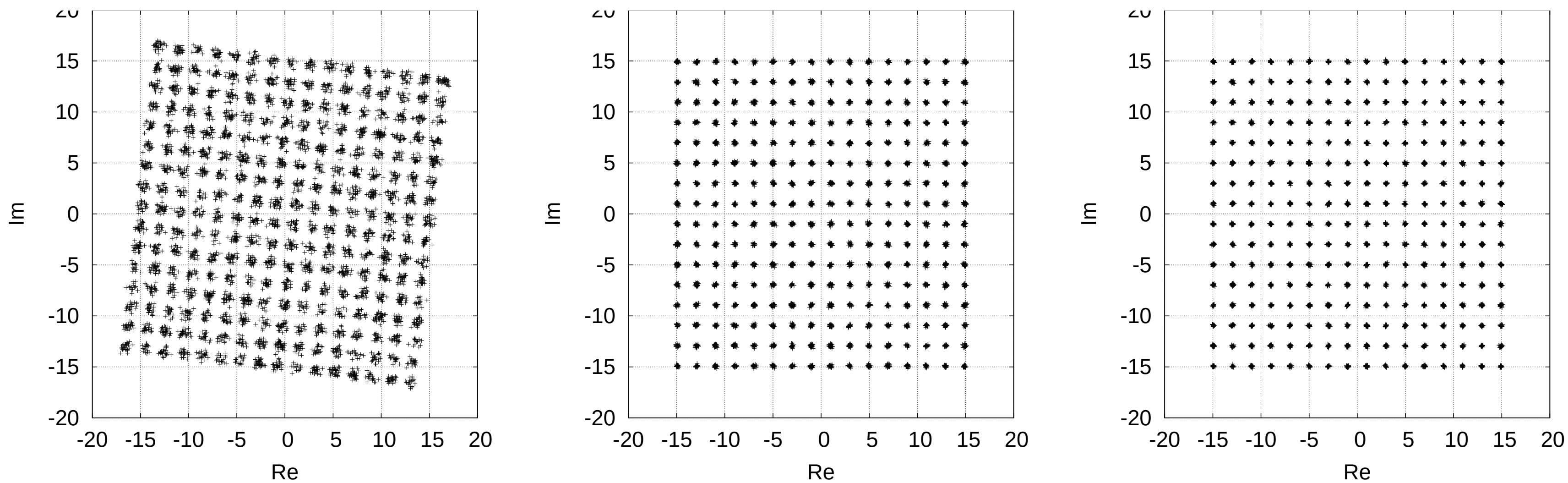
$$\mathrm{MD} = \frac{\sum_{i=0}^{N_f-1} |f_i| - |f_{i_{\max}}|}{|f_{i_{\max}}|}$$





Signal Constellation (Fixed SISO Channel)

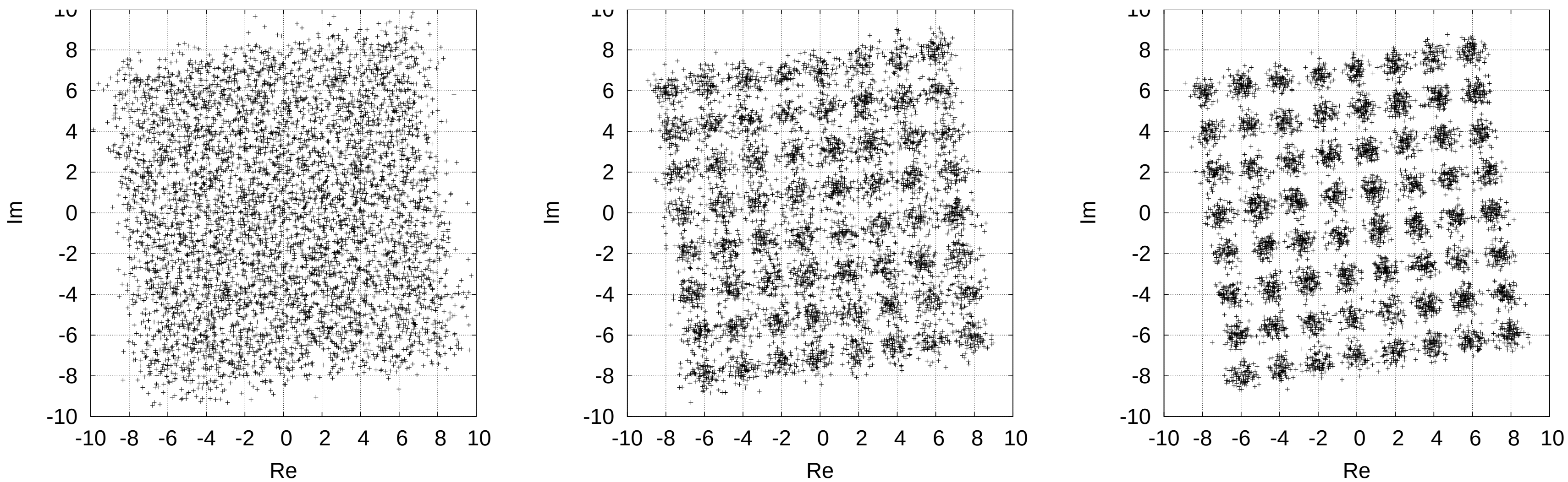
Equalizer output signal constellations after convergence (a) CMA, (b) CMA+DD, and (c) CMA+SDD



(a) (b) (c)

64-QAM Fading SISO Channel (CMA)

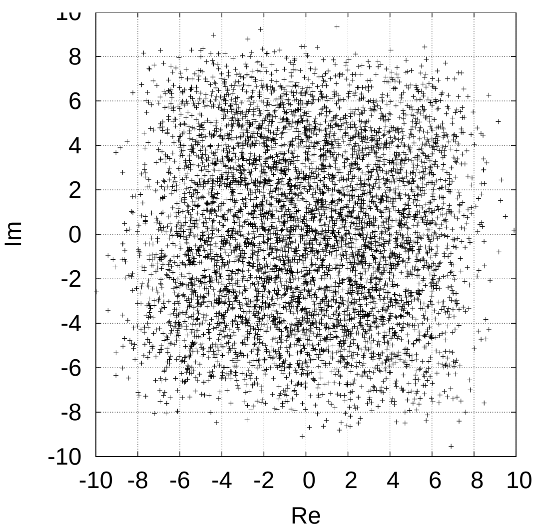
CMA equalizer output signal constellations: (a) after adaptation of 20000 symbols, (b) after adaptation of 25000 symbols, and (c) after adaptation of 30000 symbols. 6000 T_s-spaced samples were used in showing signal constellation with continuous adaptation



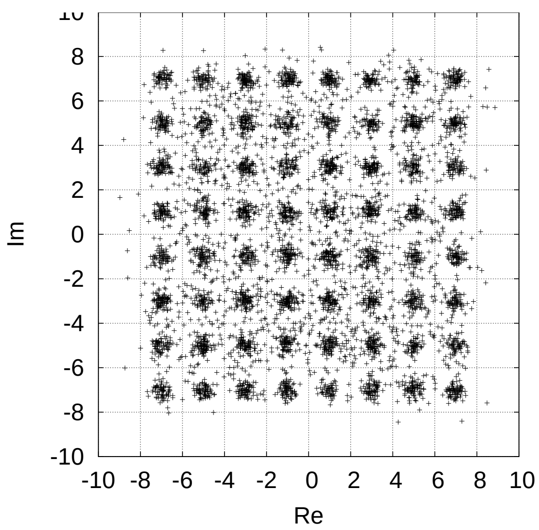
(a) (b) (c)

64-QAM Fading SISO Channel (CMA+DD)

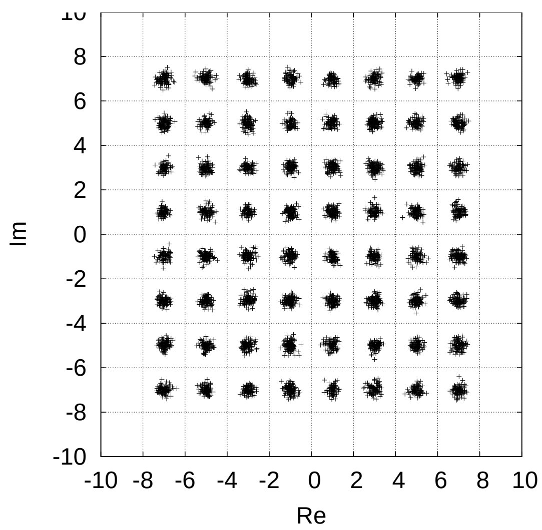
CMA+DD equalizer output signal constellations: (a) after adaptation of 15000 symbols, (b) after adaptation of 20000 symbols, and (c) after adaptation of 25000 symbols. 6000 T_s-spaced samples were used in showing signal constellation with continuous adaptation



(a)



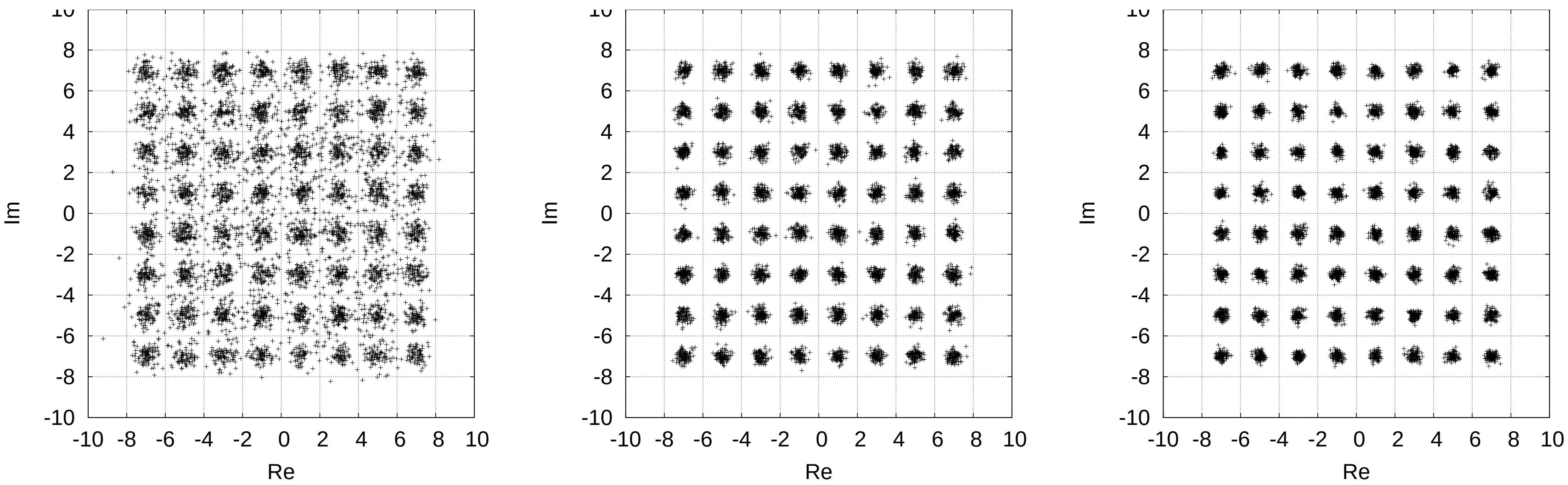
(b)



(c)

64-QAM Fading SISO Channel (CMA+SDD)

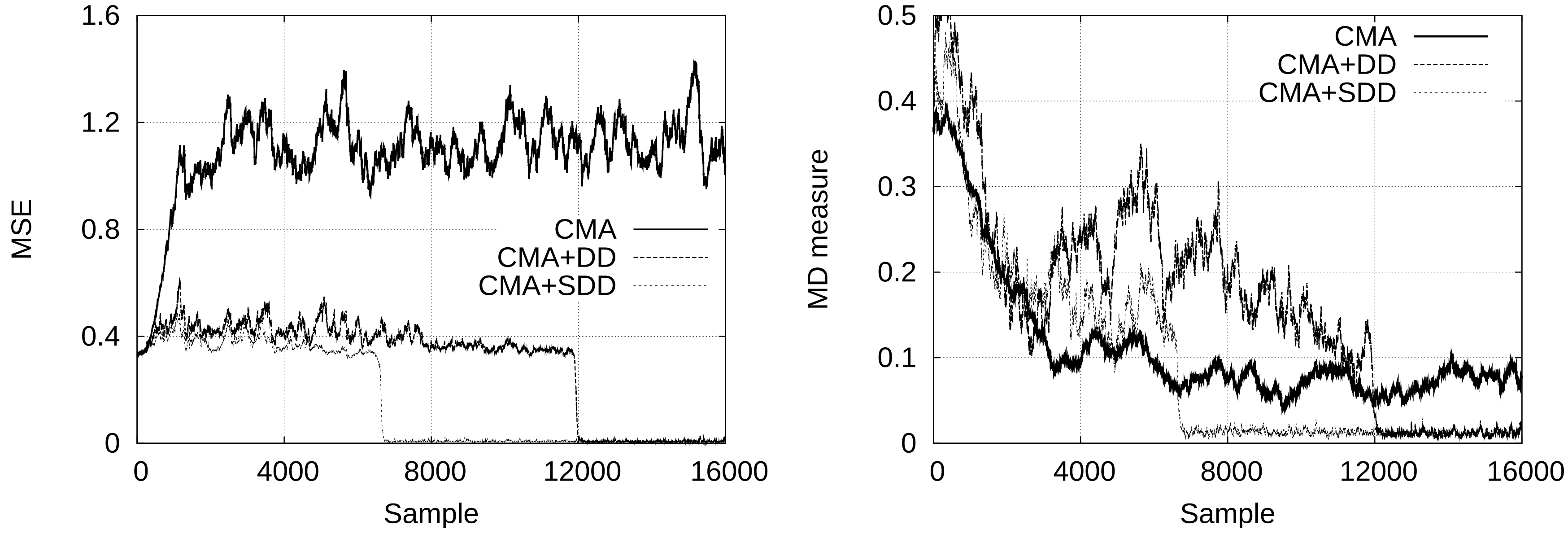
CMA+SDD equalizer output signal constellations: (a) after adaptation of 15000 symbols, (b) after adaptation of 20000 symbols, and (c) after adaptation of 25000 symbols. 6000 T_s-spaced samples were used in showing signal constellation with continuous adaptation



(a) (b) (c)

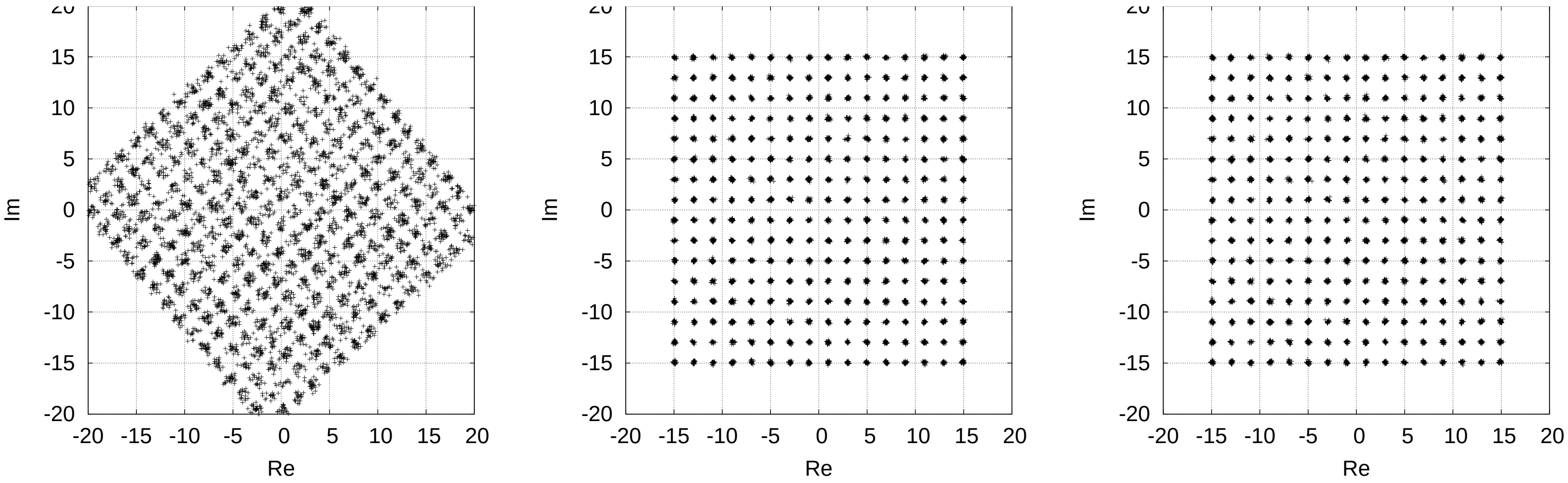
256-QAM Fixed SIMO Channel

- Single transmit antenna and four receive antennas
- $T_s/2$-spaced channels and $T_s/2$-spaced space-time equalizer



256-QAM Fixed SIMO Channel

Space-time equalizer output signal constellations after convergence: (a) CMA, (b) CMA+DD, and (c) CMA+SDD for fixed SIMO channel with 256-QAM and SNR of 40 dB



(a) (b) (c)

Conclusions

- A constant modulus algorithm aided soft decision directed scheme has been derived for low complexity blind equalization of high-order QAM channels
- Compared with an existing CMA and decision directed scheme, the proposed blind equalization scheme has simpler operational requirements and faster convergence, and achieves similar steady-state equalization performance
- Original derivation is for SISO systems, but the scheme can be extended to blind space-time equalization of SIMO and MIMO systems