Smart Beamforming for Wireless Communications: A Novel Minimum Bit Error Rate Approach

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ABSTRACT

Spatial processing with adaptive antenna array has shown real promise for substantial capacity enhancement in wireless communications. We propose a novel beamforming technique based on the minimum bit error rate (MBER) criterion. It is demonstrated that the MBER approach utilizes the system resource, the antenna elements, more intelligently than the standard minimum mean square error (MMSE) approach. Consequently, the MBER beamforming can provide significant performance gain in terms of smaller bit error rate (BER) over the MMSE beamforming.

I. INTRODUCTION

The ever-increasing demand for mobile communication capacity has motivated the needs for new technologies, such as space division multiple access, to improve spectrum utilization. One approach that has shown real promise for substantial capacity enhancement is the use of spatial processing with adaptive antenna arrays [**?**]–[**?**]. Adaptive beamforming is capable of separating signals transmitted on the same carrier frequency, provided that they are separated in the spatial domain. The beamforming processing appropriately combines the signals received by the different elements of an antenna array to form a single output. Classically, this is done by minimizing the mean square error (MSE) between the desired and actual array outputs. This has its root in the traditional beamforming employed in sonar and radar systems. However, for a communication system, it is the BER, not the MSE, that really counts. We derive a novel beamforming technique based on minimizing the system BER.

II. SYSTEM MODEL

It is assumed that the system consists of M users (sources), and each user transmits a binary phase shift keying (BPSK) signal on the same carrier frequency $\omega = 2\pi f$. The baseband signal of user i is

$$m_i(k) = A_i b_i(k), \;\; b_i(k) \in \{\pm 1\}, \;\; 1 \le i \le M, \quad (1)$$

where A_i^2 denotes user i signal power. Without the loss of generality, source 1 is the desired user and the rest of the sources are interfering users. The linear antenna array consists of L uniformly spaced elements, and signals at the L-element antenna array are

$$x_l(k) = \sum_{i=1}^{M} m_i(k) \exp\left(j\omega t_l(\theta_i)\right) + n_l(k)$$
$$= \bar{x}_l(k) + n_l(k), \;\; 1 \le l \le L, \quad (2)$$

where $t_l(\theta_i)$ is the relative time delay at element l for source i, θ_i is the direction of arrival for source i, and $n_l(k)$ is a complex-valued white Gaussian noise with zero mean and $E[|n_l(k)|^2] = 2\sigma_n^2$. The desired signal to noise ratio is defined as SNR$= A_1^2/2\sigma_n^2$, the interferer i to noise ratio is INR$_i = A_i^2/2\sigma_n^2$, and the desired signal to interferer i ratio is SIR$_i = A_1^2/A_i^2$, for $i = 2, \cdots, M$. In vector form, the array input $\mathbf{x}(k) = [x_1(k) \cdots x_L(k)]^T$ can be expressed as

$$\mathbf{x}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k) = \mathbf{P}\mathbf{b}(k) + \mathbf{n}(k) \quad (3)$$

where $E[\mathbf{n}(k)\mathbf{n}^H(k)] = 2\sigma_n^2 \mathbf{I}_L$, the system matrix $\mathbf{P} = [A_1\mathbf{s}_1 \cdots A_M\mathbf{s}_M]$, the steering vector for source i $\mathbf{s}_i = [\exp(j\omega t_1(\theta_i)) \cdots \exp(j\omega t_L(\theta_i))]^T$ and the bit vector $\mathbf{b}(k) = [b_1(k) \cdots b_M(k)]^T$. The beamformer output is

$$y(k) = \mathbf{w}^H\mathbf{x}(k) = \mathbf{w}^H\bar{\mathbf{x}}(k) + \mathbf{w}^H\mathbf{n}(k) = \bar{y}(k) + e(k) \quad (4)$$

where $\mathbf{w}$ is the complex-valued beamformer weight vector, and $e(k)$ is Gaussian with zero mean and $E[|e(k)|^2] = 2\sigma_n^2\mathbf{w}^H\mathbf{w}$. The estimate of the transmitted bit $b_1(k)$ is

$$\hat{b}_1(k) = \begin{cases} +1, & y_R(k) > 0, \\ -1, & y_R(k) \le 0, \end{cases} \quad (5)$$

where $y_R(k) = \Re[y(k)]$. The classical MMSE beamforming solution is given by $\mathbf{w}_{\rm MMSE} = \left(\mathbf{P}\mathbf{P}^H + 2\sigma_n^2\mathbf{I}_L\right)^{-1}\mathbf{p}_1$, with $\mathbf{p}_1$ being the first column of $\mathbf{P}$.

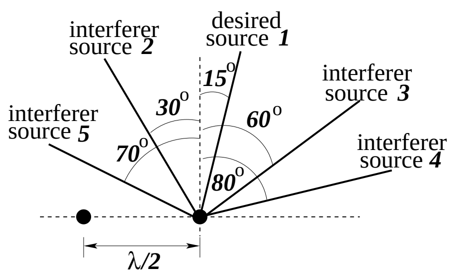


Fig. 1. Locations of the desired source and the interfering sources with respect to the two-element linear array with $\lambda/2$ element spacing, λ being the wavelength.

III. MBER Beamforming Solution

Denote the $N_b = 2^M$ possible sequences of $\mathbf{b}(k)$ as $\mathbf{b}_q$, $1 \le q \le N_b$. Further denote the first element of $\mathbf{b}_q$, corresponding to the desired user, as $b_{q,1}$. Obviously $\bar{\mathbf{x}}(k)$ only takes values from the signal state set defined as $\mathcal{X} \triangleq \{\bar{\mathbf{x}}_q = \mathbf{P}\mathbf{b}_q,\ 1 \le q \le N_b\}$. Similarly, $\bar{y}(k)$ takes values from the set $\mathcal{Y} \triangleq \{\bar{y}_q = \mathbf{w}^H \bar{\mathbf{x}}_q,\ 1 \le q \le N_b\}$. Thus, $\bar{y}_R(k)$ can only take values from the set

$$\mathcal{Y}_R \triangleq \{\bar{y}_{R,q} = \Re[\bar{y}_q],\ \ 1 \le q \le N_b\} \tag{6}$$

which can be divided into the two subsets

$$\mathcal{Y}_R^{(\pm)} \triangleq \{\bar{y}_{R,q}^{(\pm)} \in \mathcal{Y}_R :\ b_1(k) = \pm 1\}. \tag{7}$$

The conditional probability density function (p.d.f.) of $y_R(k)$ given $b_1(k) = +1$ is

$$p(y_R|+1) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} \frac{1}{\sqrt{2\pi\sigma_n^2 \mathbf{w}^H \mathbf{w}}} \exp\left(-\frac{\left(y_R - \bar{y}_{R,q}^{(+)}\right)^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}}\right) \tag{8}$$

where $\bar{y}_{R,q}^{(+)} \in \mathcal{Y}_R^{(+)}$ and $N_{sb} = N_b/2$ is the number of the points in $\mathcal{Y}_R^{(+)}$. Thus the BER is given by

$$P_E(\mathbf{w}) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} Q\left(g_{q,+}(\mathbf{w})\right) \tag{9}$$

where

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty \exp\left(-\frac{v^2}{2}\right) dv \tag{10}$$

and

$$g_{q,+}(\mathbf{w}) = \frac{\mathrm{sgn}(b_{q,1}) \bar{y}_{R,q}^{(+)}}{\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}} = \frac{\mathrm{sgn}(b_{q,1}) \Re[\mathbf{w}^H \bar{\mathbf{x}}_q^{(+)}]}{\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}}. \tag{11}$$

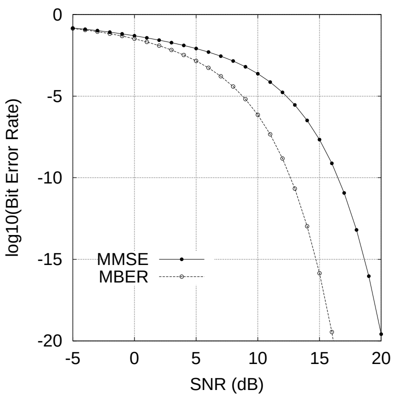
The MBER beamforming solution is then defined as

$$\mathbf{w}_{\mathrm{MBER}} = \arg\min_{\mathbf{w}} P_E(\mathbf{w}). \tag{12}$$

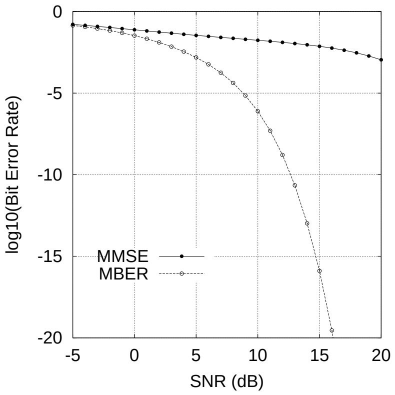
The gradient of $P_E(\mathbf{w})$ with respect to $\mathbf{w}$ is

$$\nabla P_E(\mathbf{w}) = \frac{1}{2N_{sb}\sqrt{2\pi}\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}} \sum_{q=1}^{N_{sb}} \exp\left(-\frac{\left(\bar{y}_{R,q}^{(+)}\right)^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}}\right) \times \mathrm{sgn}(b_{q,1}) \left(\frac{\bar{y}_{R,q}^{(+)} \mathbf{w}}{\mathbf{w}^H \mathbf{w}} - \bar{\mathbf{x}}_q^{(+)}\right). \tag{13}$$

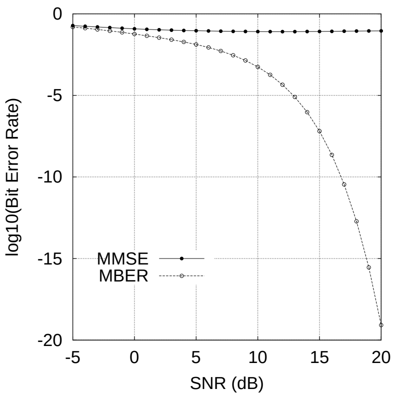
The optimization problem (**??**) can be solved for iteratively using a conjugated gradient algorithm with a resetting of the search direction periodically to the negative gradient. Note that the BER is invariant to a positive scaling of $\mathbf{w}$. Similarly, the BER can be calculated alternatively using $\mathcal{Y}_R^{(-)}$.



(a) SNR=INR$_i$ for $i = 2, 3, 4, 5$.



(b) SNR=INR$_i$ for $i = 3, 4, 5$, and INR$_2$ =SNR+ 6 dB.



(c) INR$_i$ =SNR+ 6 dB for $i = 2, 3, 4, 5$.

Fig. 2. Comparison of bit error rate performance.

IV. Simulation Study

The example consisted of five sources and a two-element antenna array. Fig. **??** shows the locations of the desired source and the interfering sources graphically. Figs. **??** compares the BER performance of the MBER solution with that of the MMSE solution under three different conditions: (a) the desired user and all the four interfering sources have equal power, (b) the desired user and the interfering sources $3, 4, 5$ have equal power, but the interfering source 2 has 6 dB more power than the desired user, and (c) all the four inter-

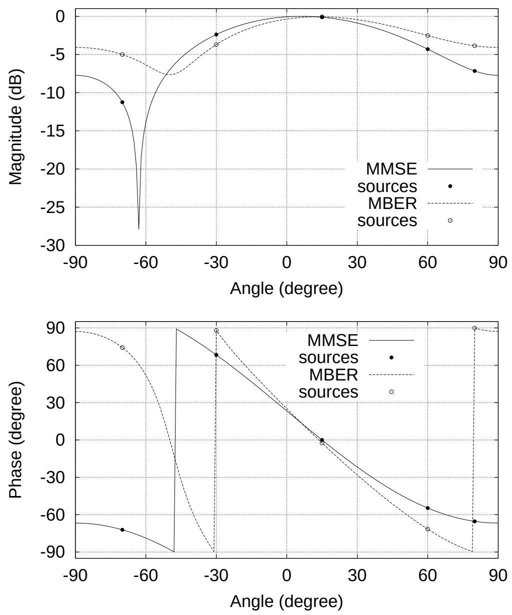


Fig. 3. Comparison of beam patterns. SNR=INR$_i$ $= 10$ dB, $i = 2, 3, 4, 5$.

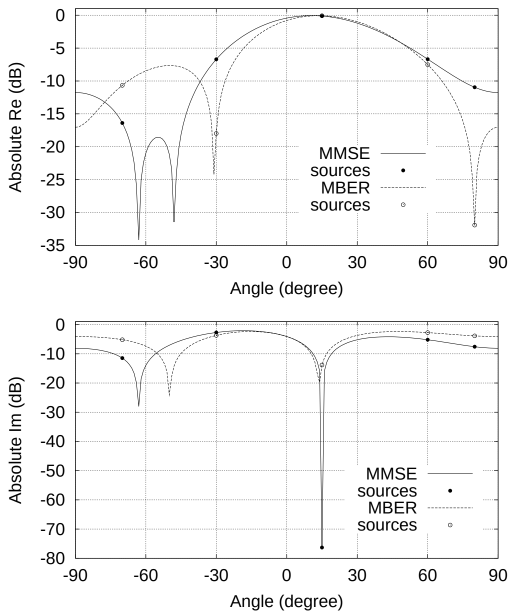


Fig. 4. Alternative display of beam patterns in Fig. **??**.

fering sources have 6 dB more power than the desired user.

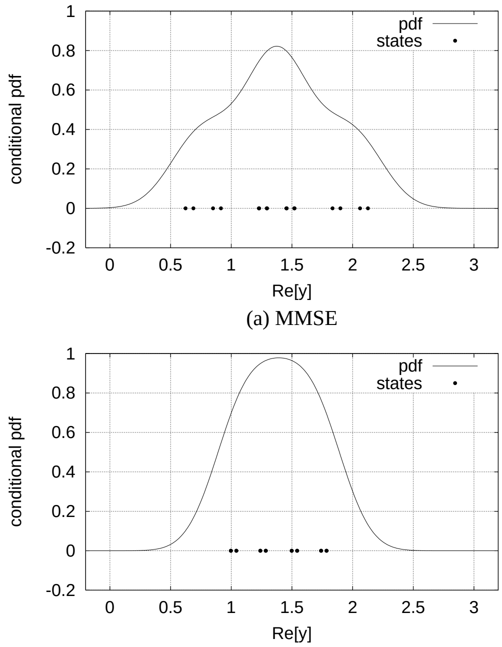
The discrete Fourier transform (DFT) of the beamformer weights or beam pattern

$$F(\theta) = \sum_{l=1}^{L} w_l \exp\left(-j\omega t_l(\theta)\right) \qquad (14)$$

describes the response of the beamformer to the source arriving at angle θ. Fig. **??** compares the DFT of the MBER beamformer with that of the MMSE beamformer under the condition SNR=INR$_i$ $= 10$ dB for $i = 2, 3, 4, 5$, where $F(\theta)$ has been normalized. In traditional beamforming, the magnitude of $F(\theta)$ is used to judge the performance of a beamformer. It appears that the MMSE beamformer has better magnitude response than the MBER beamformer. Specifically, at the four angles for the four interfering sources, the MMSE solution has better magnitude responses at -70°, 60° and 80°, and a lightly inferior magnitude response at -30°, compared with the MBER solution. However, magnitude response along can be misleading. At the four angles for the four interfering sources, the phase responses of the MBER solution are much closer to $\pm\frac{\pi}{2}$ than the MMSE solution, which give rise to a much better response of $y_R(k) = \Re[y(k)]$. Thus the MBER solution has a better capacity to "cancel" interfering signals.

It is interesting to see in more details how the two beamformers utilize the antenna array resource (the beamformer weights) by examining the real and imaginary parts of the beam pattern. Fig. **??** depicts $|\Re[F(\theta)]|$ and $|\Im[F(\theta)]|$ of the two beam patterns, under the same condition of Fig. **??**. Note that the BER depends only on the real part of the beamformer output. The MBER solution concentrates the resource to achieve a better response $|\Re[F(\theta)]|$ and does not care much about $|\Im[F(\theta)]|$. The MMSE solution on the other hand has a null at $|\Im[F(15^\circ)]|$, 15° being the arrival angle of the desired user. This is obviously crucial in minimizing the MSE $E[|b_1(k) - y(k)|^2]$, as $b_1(k)$ is real-valued. However, this is irrelevant to the system BER performance. Clearly, the MBER beamformer uses the system resource more cleverly. The conditional p.d.f. (**??**) is the best indicator for the BER performance of the beamformer. Fig. **??** compares the conditional p.d.f. of the MBER solution with that of the MMSE one, under the same condition of Fig. **??**. In Fig. **??**, the beamformer weight vector has been normalized to a unit length, so that the BER is mainly determined by the minimum distance of the subset $\mathcal{Y}_R^{(+)}$ to the decision threshold $y_R = 0$.

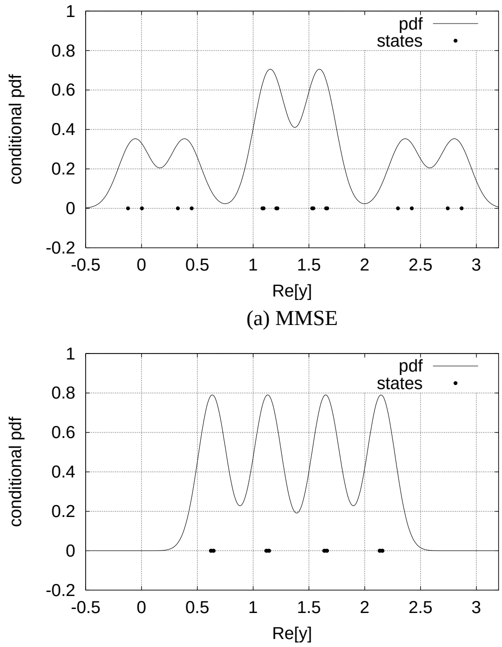
Under the condition given in Fig. **??** (c), the MMSE beamformer has an extremely poor BER performance. The reason for this is now investigated. Given SNR= 15 dB and INR$_i$ =SNR+6 dB for $i = 2, 3, 4, 5$, the beam patterns for the MMSE and MBER beamformers are in fact very similar to those shown in Fig. **??**, which can explain why the MBER solution has better BER performance but cannot explain why the MMSE solution should break down. By examining the conditional p.d.f.s of the two beamformers, as illustrated in Fig. **??**, it becomes clearly why the MMSE solution has a high BER floor. For the MMSE solution, $\mathcal{Y}_R^{(-)}$ and $\mathcal{Y}_R^{(+)}$ are linearly inseparable. There are $N_{sb} = 16$ points in $\mathcal{Y}_R^{(+)}$. One of them is on the wrong side of the decision boundary $y_R = 0$ and another point is right on $y_R = 0$. Fig. **??** (c)



(a) MMSE

(b) MBER

Fig. 5. Conditional probability density function of beamformer given $b_1(k) = +1$. SNR=INR$_i$ $= 10$ dB, $i = 2, 3, 4, 5$.



(a) MMSE

(b) MBER

Fig. 6. Conditional probability density function of beamformer given $b_1(k) = +1$. SNR= 15 dB and INR$_i$ =SNR+6 dB for $i = 2, 3, 4, 5$.

also indicates that the MBER solution is robust to the near-far effect, and this is further confirmed by the results shown in Fig. **??**.

V. Conclusions

A novel MBER beamforming solution has been derived. It has been demonstrated that the MBER beamformer utilizes the system resource more intelligently than the standard MMSE beamformer and, consequently, can achieve a better performance in terms of a smaller BER. The results also suggest that the MBER solution is robust to the near-far effect. The adaptive implementation of the MBER beamformer is not addressed in the current paper. However, it is well-known that the theoretical MMSE beamforming solution can adaptively be implemented using temporal reference techniques, such as the least mean square (LMS) algorithm. Similarly, the theoretical MBER beamforming solution can adaptively be implemented using a LMS-style stochastic gradient algorithm called the least bit error rate algorithm [**?**],[**?**]. Currently, we are also working on the extension of the MBER beamforming to other modulation schemes.

References

[1] J.H. Winters, J. Salz and R.D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. Communications*, Vol.42, No.2, pp.1740–1751, February/March/April 1994.

[2] J. Litva and T. K.Y. Lo, *Digital Beamforming in Wireless Communications*. London: Artech House, 1996.

[3] L. C. Godara, "Applications of antenna arrays to mobile communications, Part I: Performance improvement, feasibility, and system considerations," *Proc. IEEE*, Vol.85, No.7, pp.1031–1060, 1997.

[4] J.S. Blogh and L. Hanzo, *Third Generation Systems and Intelligent Wireless Networking – Smart Antenna and Adaptive Modulation*. Chichester: John Wiley, 2002.

[5] B. Mulgrew and S. Chen, "Adaptive minimum-BER decision feedback equalisers for binary signalling," *Signal Processing*, Vol.81, No.7, pp.1479–1489, 2001.

[6] S. Chen, A.K. Samingan, B. Mulgrew and L. Hanzo, "Adaptive minimum-BER linear multiuser detection for DS-CDMA signals in multipath channels," *IEEE Trans. Signal Processing*, Vol.49, No.6, pp.1240–1247, 2001.

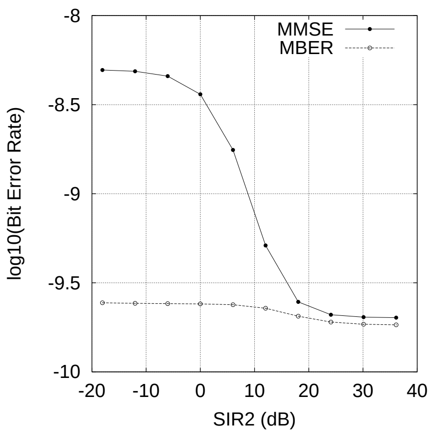


Fig. 7. Influence of the near-far effect to bit error rate performance. SNR= 10 dB, SIR$_i$ $= 24$ dB for $i = 3, 4, 5$.