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Downlink MBER Beamforming Transmitter Based on Uplink MBER Beamforming Receiver for TDD-SDMA MIMO Systems

Sheng Chen and Lie-Liang Yang

School of Electronics and Computer Science
University of Southampton, Southampton SO17 1BJ, UK
sqc@ecs.soton.ac.uk   lly@ecs.soton.ac.uk

Abbreviations

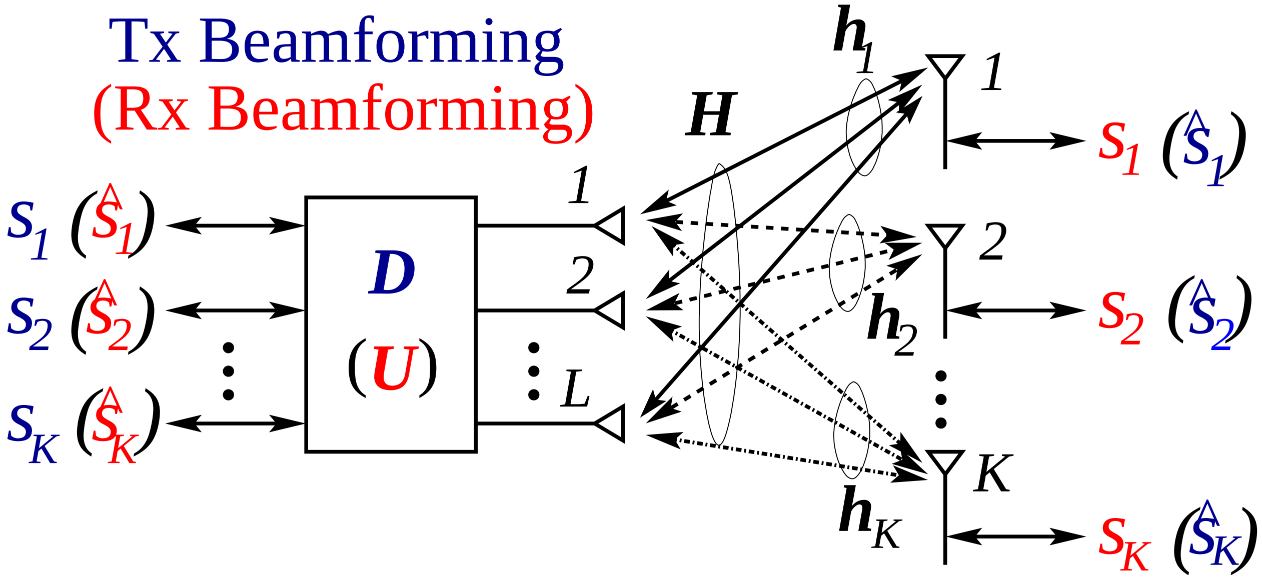
- MIMO → multiple-input multiple-output
- SDMA → space-division multiple-access
- TDD → time division duplexing
- BS / MT → base station / mobile terminal
- MUI → multiuser interference
- MUD / MUT → multiuser detection / multiuser transmission
- Tx / Rx → transmit / receive
- MMSE → minimum mean square error
- MBER → minimum bit error rate
- CSI → channel state information

Motivations

- ❑ In uplink, BS receiver is capable of implementing sophisticated MUD, e.g. Rx beamforming, to mitigate MUI
- ❑ In downlink, simple MT receivers are unable to perform sophisticated cooperative MUD
- ❑ BS can carry Tx preprocessing for mitigating MUI, leading to MUT, e.g. Tx beamforming, provided that BS has downlink CSI
- ❑ For TDD system, there exists dual relationship between MUD and MUT, owing to channel reciprocity of uplink and downlink
- ❑ Since BS has to implement MUD, it may readily implement downlink MUT based on uplink MUD solution with no computational cost

System Model

- TDD-SDMA MIMO: BS with L antennas ↔ K single-antenna MTs



- ☆ Uplink channel $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_K]$, and downlink is reciprocal
- ☆ Uplink and downlink Tx symbols both denoted as $\mathbf{s} = [s_1\ s_2 \cdots s_K]^T$
- ☆ Uplink noise $\mathbf{n}_U$ with $E[\mathbf{n}_U \mathbf{n}_U^H] = 2\sigma_U^2 \mathbf{I}_L$, and downlink noise $\mathbf{n}_D$ with $E[\mathbf{n}_D \mathbf{n}_D^H] = 2\sigma_D^2 \mathbf{I}_K$, where $\sigma_U^2 = \sigma_D^2$

MUD and MUT

- ☞ Uplink received signal vector

$$\mathbf{x}_U = \mathbf{H}\,\mathbf{s} + \mathbf{n}_U$$

- ☞ BS's MUD decision variable vector

$$\mathbf{y}_U = \mathbf{U}^H \mathbf{x}_U = \mathbf{U}^H \mathbf{H}\,\mathbf{s} + \mathbf{U}^H \mathbf{n}_U$$

with MUD coefficient matrix given by $\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2 \cdots \mathbf{u}_K]$

- ☛ Downlink MUT preprocessing matrix at BS

$$\mathbf{D} = [\mathbf{d}_1\ \mathbf{d}_2 \cdots \mathbf{d}_K]$$

- ☛ Downlink receive signal vector or decision variable vector at K MTs

$$\mathbf{y}_D = \mathbf{H}^T \mathbf{D}\,\mathbf{s} + \mathbf{n}_D$$

Duality between MUD and MUT

❑ Existing duality between MUD and MUT: Given $\sigma_U^2 = \sigma_D^2$,

$$\mathbf{D} = \mathbf{U}^* \mathbf{\Lambda}$$

where $\mathbf{\Lambda} = \text{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_K\}$ for transmit power constraint, and a simple scheme is $\lambda_k = 1/\|\mathbf{u}_k\|$, $1 \le k \le K$

☞ Conventional MUD and MUT designs are based on MMSE criteria

☞ Imply $L \geq K$ full rank systems

❑ We extend this duality to more advanced designs

✰ Specifically, for MBER MUD and MUT designs, duality holds even for $L < K$ rank-deficient systems

✰ Significance: MBER MUT design is expensive, and BS can directly implement MBER MUT based on MBER MUD solution with no cost

MBER MUD Design

- ❏ For notational simplicity, restrict to BPSK. Then sufficient statistics are

$$\Re[\mathbf{y}_U] = \Re[\mathbf{U}^H \mathbf{H}\,\mathbf{s}] + \Re[\mathbf{U}^H \mathbf{n}_U]$$

- ❏ Marginal PDFs of $\Re[y_{U,k}]$, $1 \le k \le K$, are Gaussian distributed with

$$\text{mean} \quad E\left[\Re[y_{U,k}]\right] = \Re[\mathbf{u}_k^H \mathbf{H}\,\mathbf{s}]$$

$$\text{variance} \quad \mathrm{Var}\left[\Re[y_{U,k}]\right] = \|\mathbf{u}_k\|^2 \sigma_U^2$$

- ❏ Hence BER of MUD with detector weight matrix $\mathbf{U}$ is

$$P_{Rx}(\mathbf{U}) = \frac{1}{K N_s} \sum_{k=1}^{K} \sum_{q=1}^{N_s} Q\left(\frac{\mathrm{sgn}(s_k^{(q)}) \Re[\mathbf{u}_k^H \mathbf{H} \mathbf{s}^{(q)}]}{\|\mathbf{u}_k\| \sigma_U} \right)$$

  - ☞ $Q(\bullet)$ is Gaussian error function, $N_s = 2^K$ is number of legitimate symbol vectors $\mathbf{s}^{(q)}$, $1 \le q \le N_s$, and $s_k^{(q)}$ kth element of $\mathbf{s}^{(q)}$
  - ☞ User k BER only depends on $\mathbf{u}_k$

MBER MUD (continue)

- ❑ MBER MUD solution $\mathbf{U}_{\text{MBER}} = \left[\mathbf{u}_{\text{MBER},1} \ \mathbf{u}_{\text{MBER},2} \cdots \mathbf{u}_{\text{MBER},K}\right]$ is

$$\mathbf{U}_{\text{MBER}} = \arg\min_{\mathbf{U}} P_{Rx}(\mathbf{U})$$

  - ☞ BER is invariant to the length of $\mathbf{u}_k$ → normalise $\mathbf{u}_k$ to unit-length $\|\mathbf{u}_k\| = 1$
  - ☞ Gradient-based numerical optimisation algorithm to obtain $\mathbf{U}_{\text{MBER}}$

- ❑ **Definition: E-optimum** – MBER solution $\mathbf{u}_{\text{MBER},k}$ to $\mathbf{u}_k$ is egocentric-optimum
  - ☛ self-centred, i.e. only concerned with user k, without regarding the effect on other users

- ❑ **Definition: O-optimum** – All column vectors $\mathbf{u}_{\text{MBER},k}$, $1 \leq k \leq K$, are optimum in some sense (E-optimum) → $\mathbf{U}_{\text{MBER}}$ is overall-optimum

MBER MUT Design

- ❑ Sufficient statistics are

$$\Re[\mathbf{y}_D] = \Re[\mathbf{H}^T \mathbf{D}\,\mathbf{s}] + \Re[\mathbf{n}_D]$$

- ❑ Marginal PDFs of $\Re[y_{D,k}]$, $1 \le k \le K$, are Gaussian distributed with

$$\text{mean} \quad E\big[\Re[y_{D,k}]\big] = \Re[\mathbf{h}_k^T \mathbf{D}\,\mathbf{s}]$$

$$\text{variance} \quad \text{Var}\big[\Re[y_{D,k}]\big] = \sigma_D^2$$

- ❑ Hence BER of MUT with precoding weight matrix **D** is

$$P_{Tx}(\mathbf{D}) = \frac{1}{KN_s} \sum_{k=1}^{K} \sum_{q=1}^{N_s} Q\left(\frac{\text{sgn}(s_k^{(q)})\Re[\mathbf{h}_k^T \mathbf{D}\mathbf{s}^{(q)}]}{\sigma_D}\right)$$

  - ☞ User k BER depends on all column vectors of **D**

MBER MUT (continue)

- ❑ MBER MUT solution $\mathbf{D}_{\text{MBER}} = \left[\mathbf{d}_{\text{MBER},1}\ \mathbf{d}_{\text{MBER},2} \cdots \mathbf{d}_{\text{MBER},K}\right]$ is

$$\begin{aligned} \mathbf{D}_{\text{MBER}} &= \arg\min_{\mathbf{D}} P_{Tx}(\mathbf{D}) \\ \text{s.t.} & \quad \text{transmit power constraint is met} \end{aligned}$$

  - ☞ Constrained optimisation → Gradient-based sequential quadratic programming algorithm to obtain $\mathbf{D}_{\text{MBER}}$ with high complexity

- ❑ **Definition: A-optimum** – MBER solution $\mathbf{d}_{\text{MBER},k}$ to $\mathbf{d}_k$ is altruistic-optimum
  - ☛ not self-centred, also pay attention on mitigating its effects on other users

- ❑ All column vectors $\mathbf{d}_{\text{MBER},k}$, $1 \le k \le K$, are optimum in some sense (A-optimum) → $\mathbf{D}_{\text{MBER}}$ is overall-optimum

Duality Again

- Given $\mathbf{D} = \mathbf{U}^*$, $\sigma_U^2 = \sigma_D^2$ and $\|\mathbf{u}_k\| = 1$
  - ☛ Marginal PDFs of $\Re[y_{D,k}]$, $1 \le k \le K$, are Gaussian with

$$E\left[\Re[y_{D,k}]\right] = \Re[\mathbf{h}_k^H \mathbf{U}\,\mathbf{s}], \;\; \mathrm{Var}\left[\Re[y_{D,k}]\right] = \sigma_D^2$$

  - ☛ while marginal PDFs of $\Re[y_{U,k}]$, $1 \le k \le K$, are Gaussian with

$$E\left[\Re[y_{U,k}]\right] = \Re[\mathbf{u}_k^H \mathbf{H}\,\mathbf{s}], \;\; \mathrm{Var}\left[\Re[y_{U,k}]\right] = \sigma_D^2$$

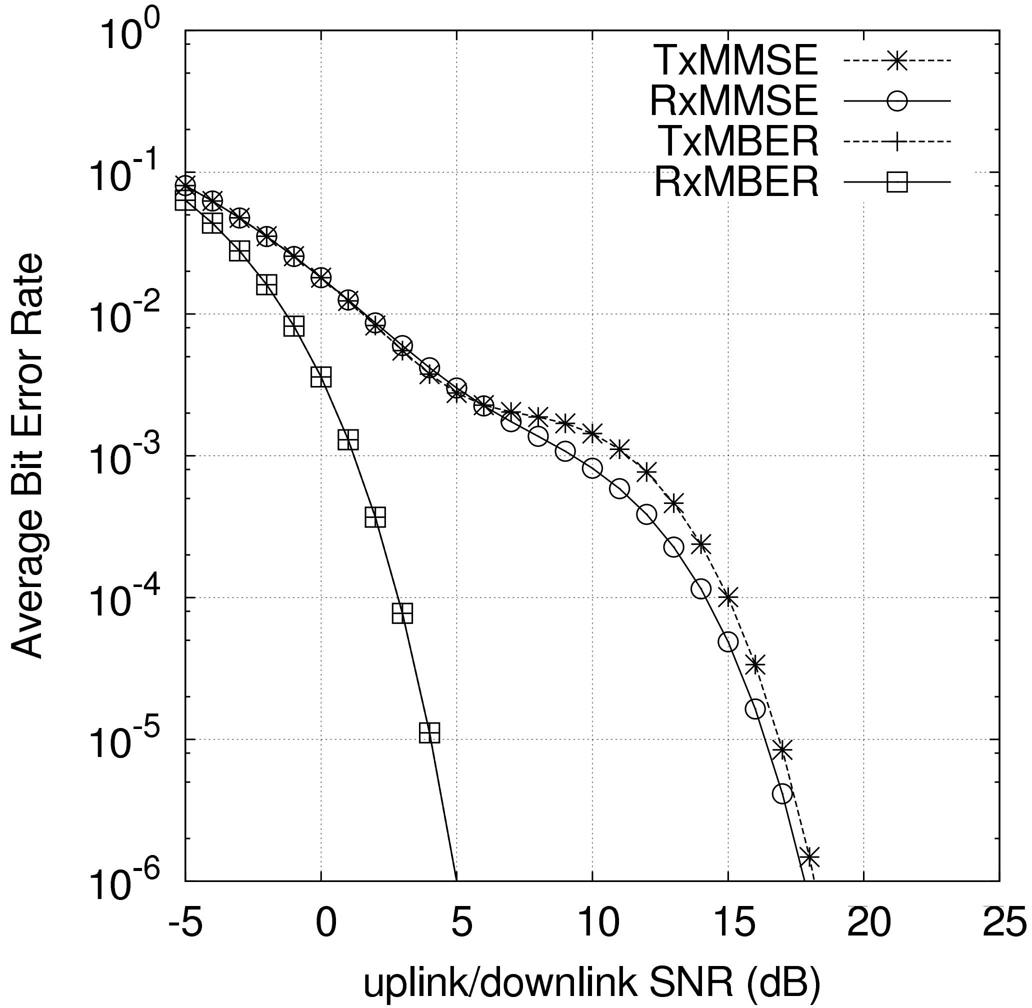
- **Proposition** An E-optimum solution in a MUD is equivalent to an A-optimum solution in the corresponding MUT

- After obtaining $\mathbf{U}_{\mathrm{MBER}}$, BS can simply set

$$\mathbf{D}_{\mathrm{MBER}} = \mathbf{U}_{\mathrm{MBER}}^*$$

to implement optimal MBER MUT with no cost

Full Rank System



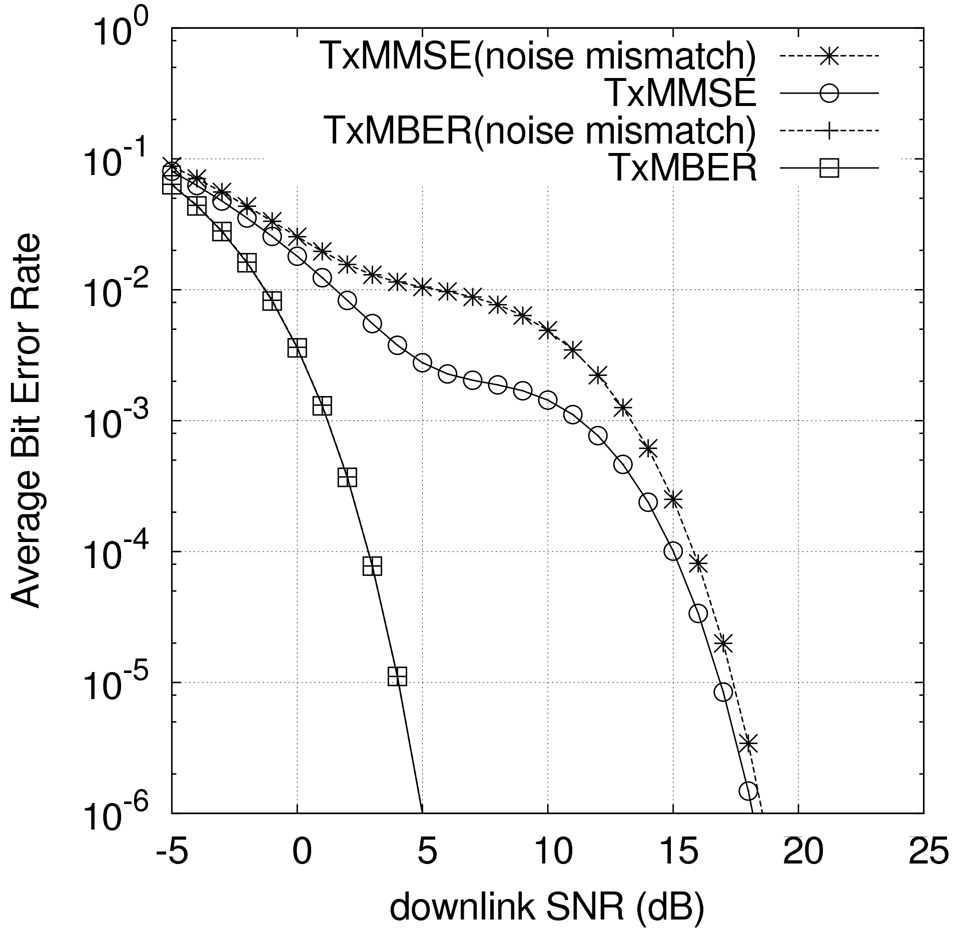
- ❑ BS has $L = 4$ antennas to support $K = 4$ single-antenna BPSK users
- ❑ BS implements MUD design $\mathbf{U}$ (MMSE or MBER)
- ❑ BS directly obtains MUT solution as

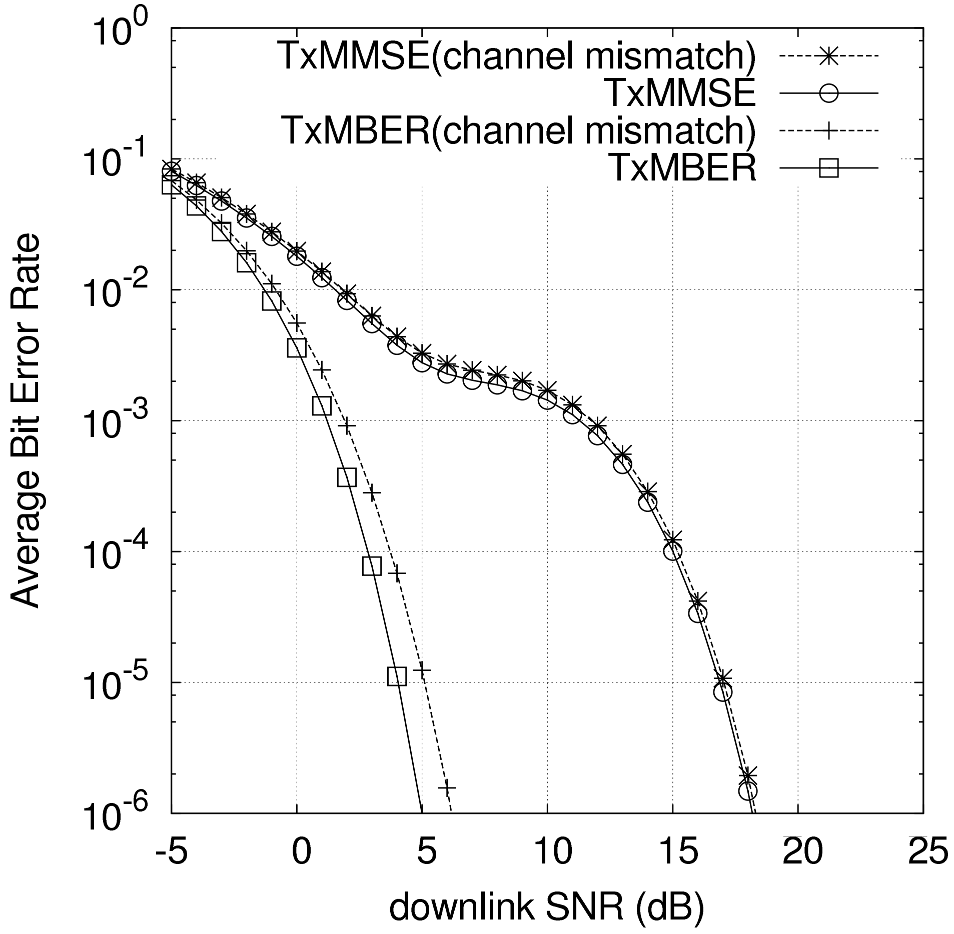
$$\mathbf{D} = \mathbf{U}^*$$

- ❑ Exact uplink and downlink channel reciprocity
- ❑ Identical uplink and downlink noise power

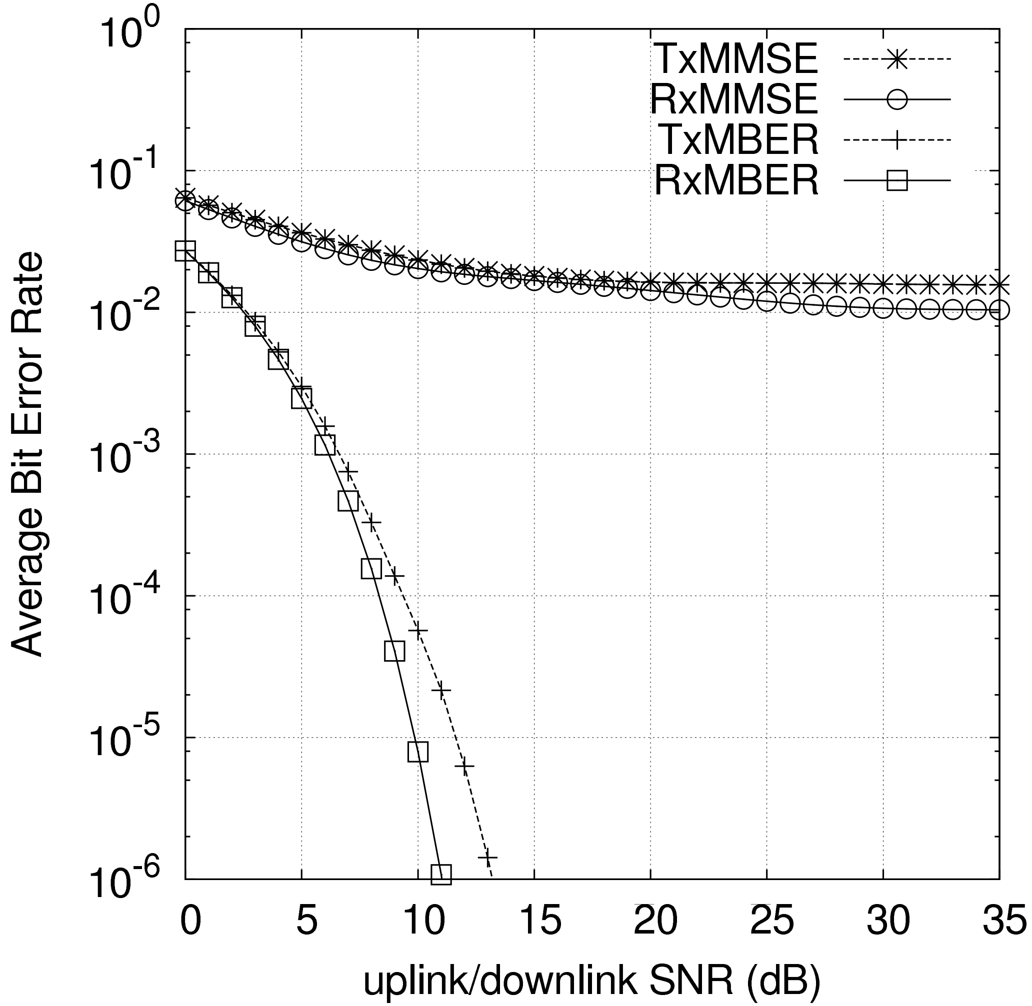
Full Rank System (continue)

- Uplink and downlink noise mismatch and channel mismatch





Rank Deficient System



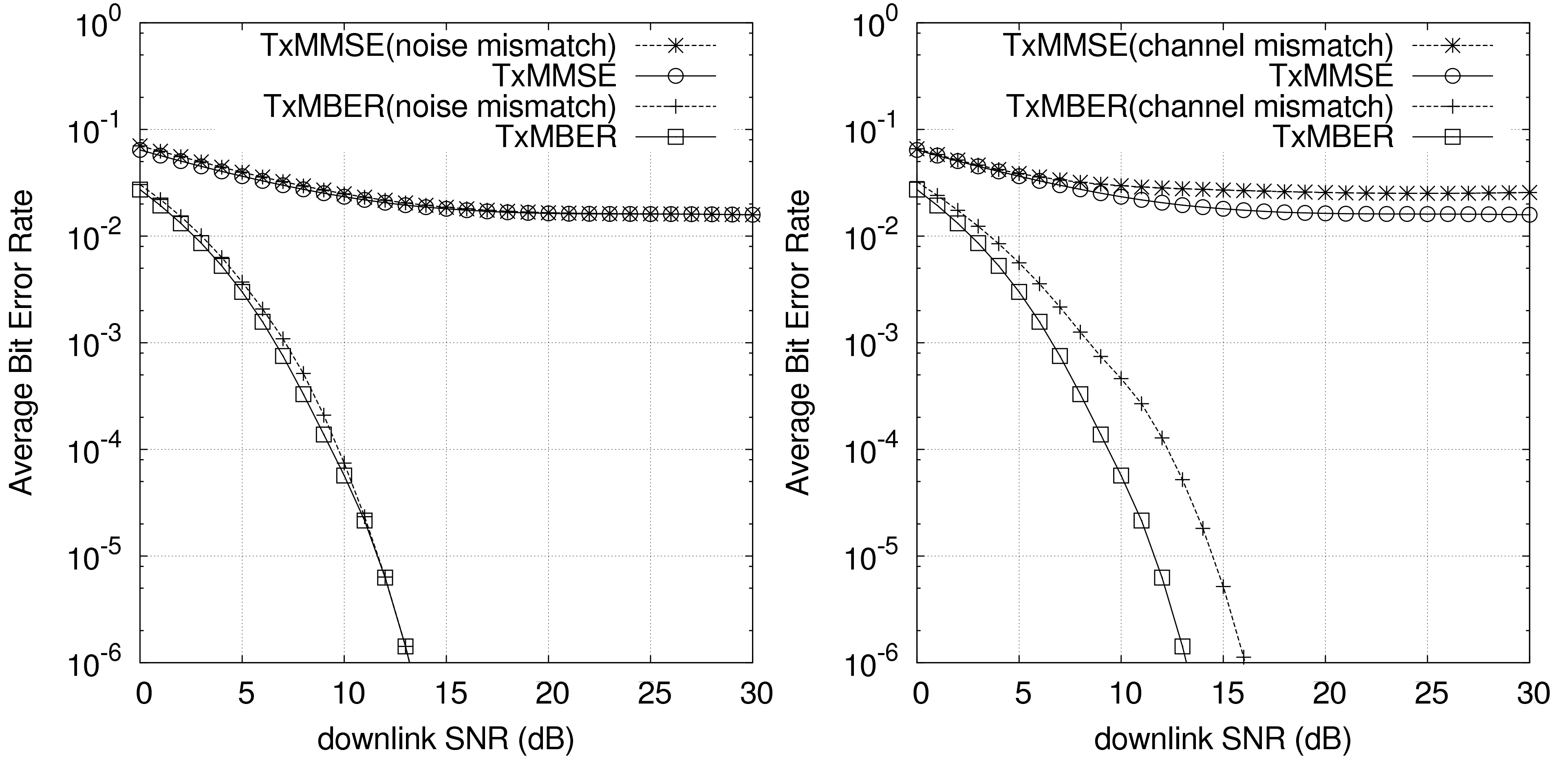
- BS has $L = 4$ antennas to support $K = 6$ single-antenna BPSK users
- BS implements MUD design $\mathbf{U}$ (MMSE or MBER)
- BS directly obtains MUT solution as

$$\mathbf{D} = \mathbf{U}^*$$

- Exact uplink and downlink channel reciprocity
- Identical uplink and downlink noise power

Rank Deficient System (continue)

- Uplink and downlink noise mismatch and channel mismatch



Conclusions

- ❑ Duality relationship between MUD and MUT can be extended to more advanced MBER designs even for rank-deficient TDD systems, where
  - ☞ Number of MTs supported is more than number of BS antennas available
- ❑ Since BS has to implement MUD anyway, it can directly obtain MUT according to this duality with no computational cost at all
  - ☞ This strategy is not overly sensitive to uplink and downlink noise or channel mismatching