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Adaptive Radial Basis Function Detector for Beamforming

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Outline

- Existing **linear** beamforming techniques, and motivations for **nonlinear** beamforming or detection

- Signal model and optimal Bayesian detection with an inherent **symmetry** property

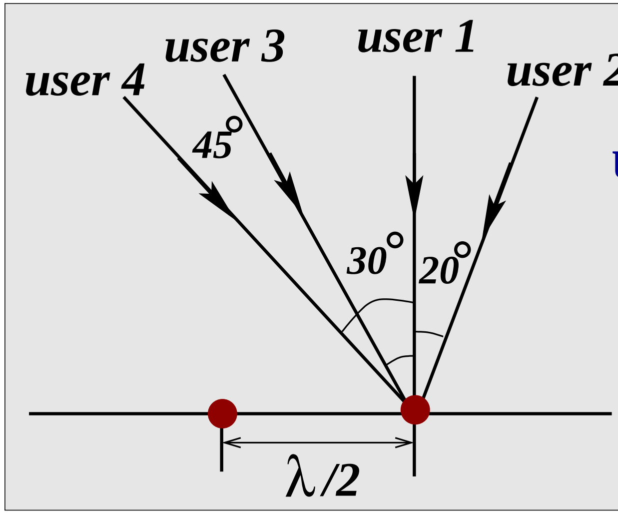
- Symmetric radial basis function network for nonlinear beamforming, and adaptive algorithms

- Simulation investigation, and performance comparison

Motivations

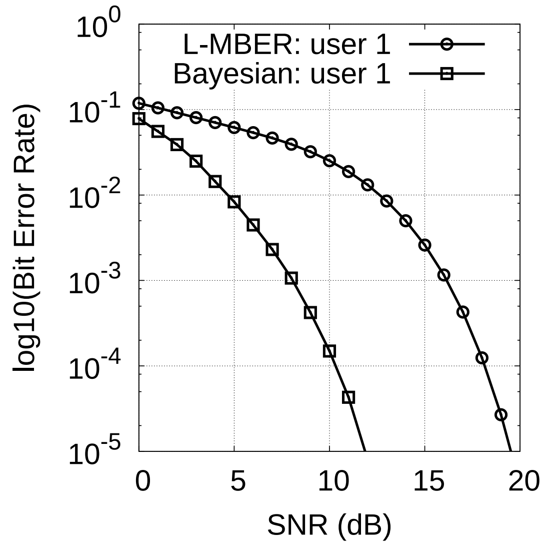
- Existing beamforming techniques are **linear**, and classical beamforming technique is based on minimum mean square error (L-MMSE)
- State-of-the-art for linear beamforming is minimum bit error rate (L-MBER) technique, and in comparsion with L-MMSE it offers
  - Better system BER performance, and larger user capacity
- Beamforming can be viewed as **classification**, which classifies received channel-impaired signal into most-likely transmitted symbol point
- In comparison with linear beamforming, **nonlinear** detection offers
  - significantly better system BER performance, and larger user capacity
  - at cost of higher complexity

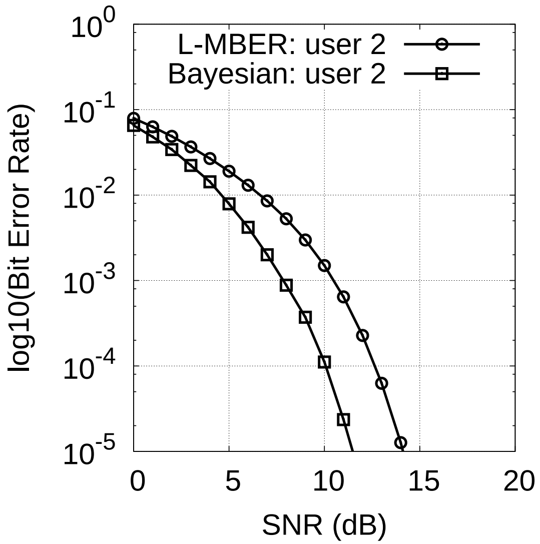
Illustration

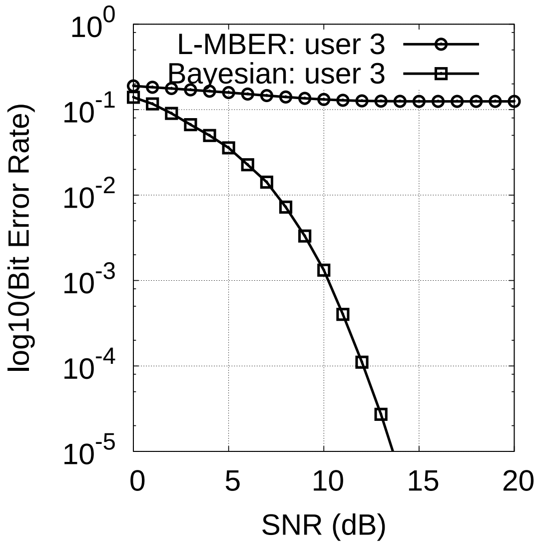


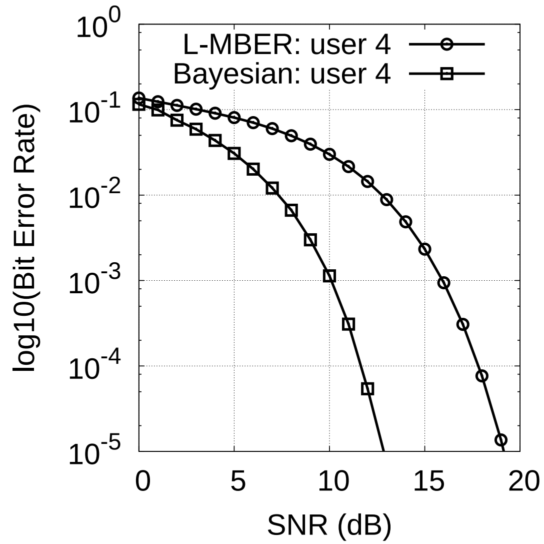
user angular positions

2−element antenna array supports 4 BPSK users









Signal Model

- M single-transmit-antenna users transmit on same carrier, receiver is equipped with L-element **antenna array**, channels are non-dispersive
- Received signal vector $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_L(k)]^T$ is

$$\mathbf{x}(k) = \mathbf{P}\,\mathbf{b}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k)$$

- $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots n_L(k)]^T$ is channel noise vector, **system matrix**

$$\mathbf{P} = [A_1\mathbf{s}_1\ A_2\mathbf{s}_2 \cdots A_M\mathbf{s}_M]$$

- $\mathbf{s}_i$ is **steering vector** of source i, A_i is i-th non-dispersive channel tap
- BPSK users $b_i(k) \in \{-1, +1\}$, $1 \le i \le M$, transmitted symbol vector

$$\mathbf{b}(k) = [b_1(k)\ b_2(k) \cdots b_M(k)]^T$$

- User 1 is **desired** user

Optimal Bayesian Detector

- Denote $N_b = 2^M$ **legitimate sequences** of $\mathbf{b}(k)$ as $\mathbf{b}_q$, $1 \le q \le N_b$, and first element of $\mathbf{b}_q$, related to **desired user**, as $b_{q,1}$
- Noiseless **channel state** $\bar{\mathbf{x}}(k)$ takes values from set

$$\bar{\mathbf{x}}(k) \in \mathcal{X} \ = \ \{\bar{\mathbf{x}}_q = \mathbf{P}\,\mathbf{b}_q, 1 \le q \le N_b\}$$

- **Optimal decision** is $\hat{b}_1(k) = \text{sgn}(y_{\text{Bay}}(k))$, with **Bayesian detector**

$$y_{\text{Bay}}(k) \ = \ f_{\text{Bay}}(\mathbf{x}(k)) \ = \ \sum_{q=1}^{N_b} \text{sgn}(b_{q,1})\beta_q e^{-\frac{\|\mathbf{x}(k)-\bar{\mathbf{x}}_q\|^2}{2\sigma_n^2}}$$

- State set can be divided into two **subsets** conditioned on value of $b_1(k)$

$$\mathcal{X}^{(\pm)} \ = \ \{\bar{\mathbf{x}}_i^{(\pm)} \in \mathcal{X}, 1 \le i \le N_{sb} : b_1(k) = \pm 1\}$$

where $N_{sb} = N_b/2 = 2^{M-1}$, and noise power is $2\sigma_n^2$

Symmetry of Bayesian Solution

- Two state subsets are **symmetric**, as

$$\mathcal{X}^{(+)} = -\mathcal{X}^{(-)}$$

- Thus Bayesian detector has **odd** symmetry, as $f_{\text{Bay}}(-\mathbf{x}(k)) = -f_{\text{Bay}}(\mathbf{x}(k))$, and it takes form

$$y_{\text{Bay}}(k) = \sum_{q=1}^{N_{sb}} \beta_q \left(e^{-\frac{\|\mathbf{x}(k)-\bar{\mathbf{x}}_q^{(+)}\|^2}{2\sigma_n^2}} - e^{-\frac{\|\mathbf{x}(k)+\bar{\mathbf{x}}_q^{(+)}\|^2}{2\sigma_n^2}} \right)$$

  since all states are equiprobable, all coefficients β_q are equal

- If system **channel matrix** $\mathbf{P}$ can be estimated, as in **uplink**, subset $\mathcal{X}^{(+)}$ can be calculated and Bayesian solution is specified

- In **downlink**, receiver only has access to desired user's training data, estimating $\mathbf{P}$ is difficult, and other adaptive means has to be adopted

Symmetric RBF Network

- Consider generic **radial basis function** network

$$y_{\text{RBF}}(k) = f_{\text{RBF}}(\mathbf{x}(k); \mathbf{w}) = \sum_{i=1}^{N_c} \theta_i \phi_i(\mathbf{x}(k))$$

- where N_c is number of RBF units, with novel **symmetric** RBF node

$$\phi_i(\mathbf{x}) = \varphi(\mathbf{x}; \mathbf{c}_i, \sigma_i^2) - \varphi(\mathbf{x}; -\mathbf{c}_i, \sigma_i^2)$$

- $\varphi(\bullet)$ is usual RBF function $\Rightarrow$ in **standard** RBF network, RBF node would simply be $\phi_i(\mathbf{x}) = \varphi(\mathbf{x}; \mathbf{c}_i, \sigma_i^2)$
- Parameter vector $\mathbf{w}$ includes all real-valued weights θ_i, complex-valued centre vectors $\mathbf{c}_i$, and positive variances σ_i^2
- Like Bayesian detector, symmetric RBF network has **odd symmetry**

$$f_{\text{RBF}}(-\mathbf{x}(k); \mathbf{w}) = -f_{\text{RBF}}(\mathbf{x}(k); \mathbf{w})$$

Nonlinear Least Bit Error Rate Algorithm

- **Theory**: given a block of training data $\{\mathbf{x}(k), b_1(k)\}$,
  - Use Parzen window estimate with Gaussian kernels to estimate probability density function of RBF network output
  - From this estimate PDF, obtain estimated detector's bit error rate
  - Minimise this estimated BER with respect to RBF parameters $\mathbf{w}$
- Adopting **stochastic** or one-sample implementation leads to NLBER

$$\begin{cases} y_{\text{RBF}}(k) & = & f_{\text{RBF}}(\mathbf{x}(k); \mathbf{w}(k-1)) \\ \mathbf{w}(k) & = & \mathbf{w}(k-1) + \frac{\mu}{\sqrt{2\pi}\rho} e^{-\frac{y^2_{\text{RBF}}(k)}{2\rho^2}} \text{sgn}(b_1(k)) \frac{\partial f_{\text{RBF}}(\mathbf{x}(k);\mathbf{w}(k-1))}{\partial \mathbf{w}} \end{cases}$$

where μ is **step size** or adaptive gain, ρ is **kernel width**

- As number of users, M, in system is usually known, number of RBF units can be set to $N_c \leq N_{sb} = 2^{M-1}$

Clustering

- Using cluster-variantion **enhanced clustering** to update RBF centres

$$\mathbf{c}_i(k) = \mathbf{c}_i(k-1) + \mu_c \mathcal{M}_i(\check{\mathbf{x}}(k))(\check{\mathbf{x}}(k) - \mathbf{c}_i(k-1))$$

where $\check{\mathbf{x}}(k) = \mathrm{sgn}(b_1(k))\mathbf{x}(k)$, and μ_c is **step size**

- **Membership function** $\mathcal{M}_i(\mathbf{x})$ is defined as

$$\mathcal{M}_i(\mathbf{x}) = \begin{cases} 1, & \text{if } \bar{v}_i \|\mathbf{x} - \mathbf{c}_i\|^2 \leq \bar{v}_j \|\mathbf{x} - \mathbf{c}_j\|^2, \ \forall j \neq i \\ 0, & \text{otherwise} \end{cases}$$

- **Cluster variations** $\bar{v}_i$ are update with rule

$$\bar{v}_i(k) = \mu_v \bar{v}_i(k-1) + (1 - \mu_v)\mathcal{M}_i(\check{\mathbf{x}}(k)) \|\check{\mathbf{x}}(k) - \mathbf{c}_i(k-1)\|^2$$

where μ_v is a constant slightly less than 1.0, e.g. $\mu_v = 0.995$, and initial variations $\bar{v}_i(0)$ are set to same small number

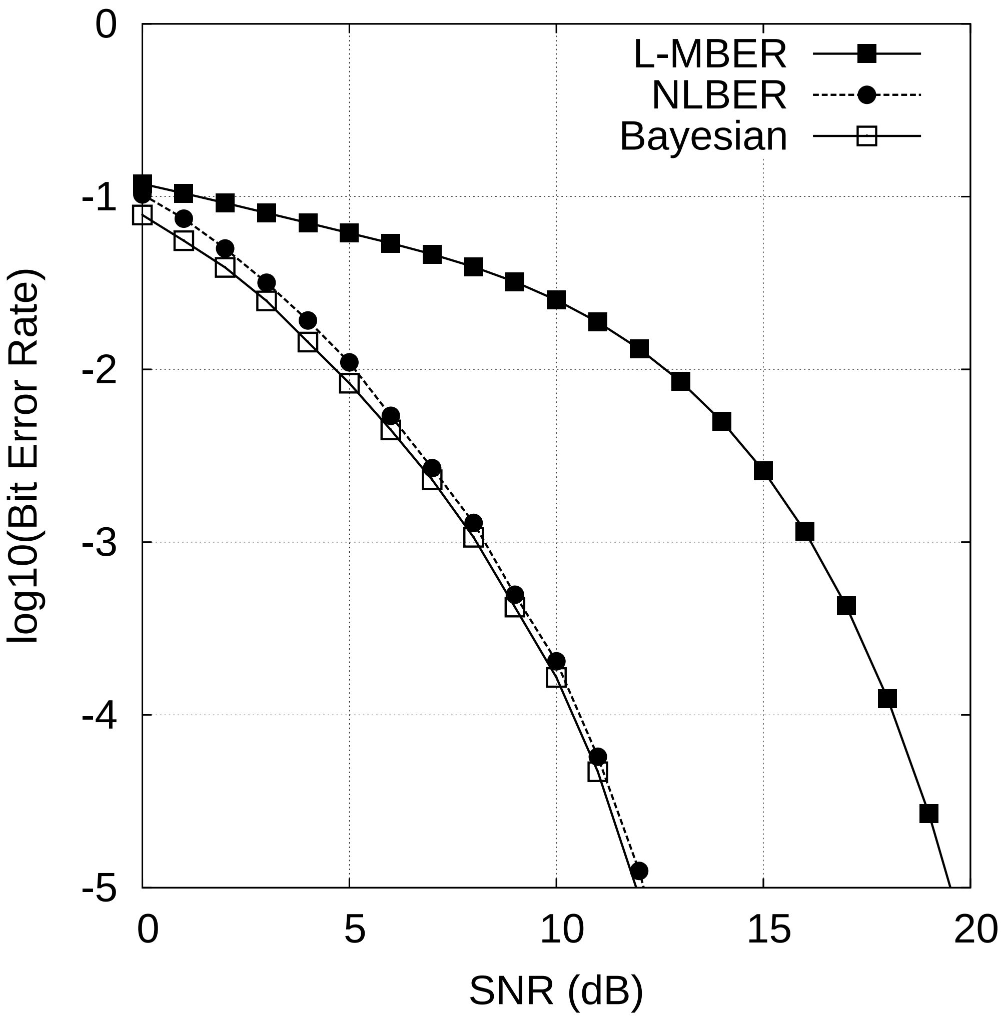
Simulation Example One

- 2-element array with half wavelength spacing, four equal-power BPSK users
- Location of users in terms of **angle of arrival**

user	1	2	3	4
AOA	$0°$	$20°$	$-30°$	$-45°$

- **Symmetric** RBF network has $N_c = N_{sb} = 8$ symmetric RBF units
- **Nonlinear least bit error rate** algorithm

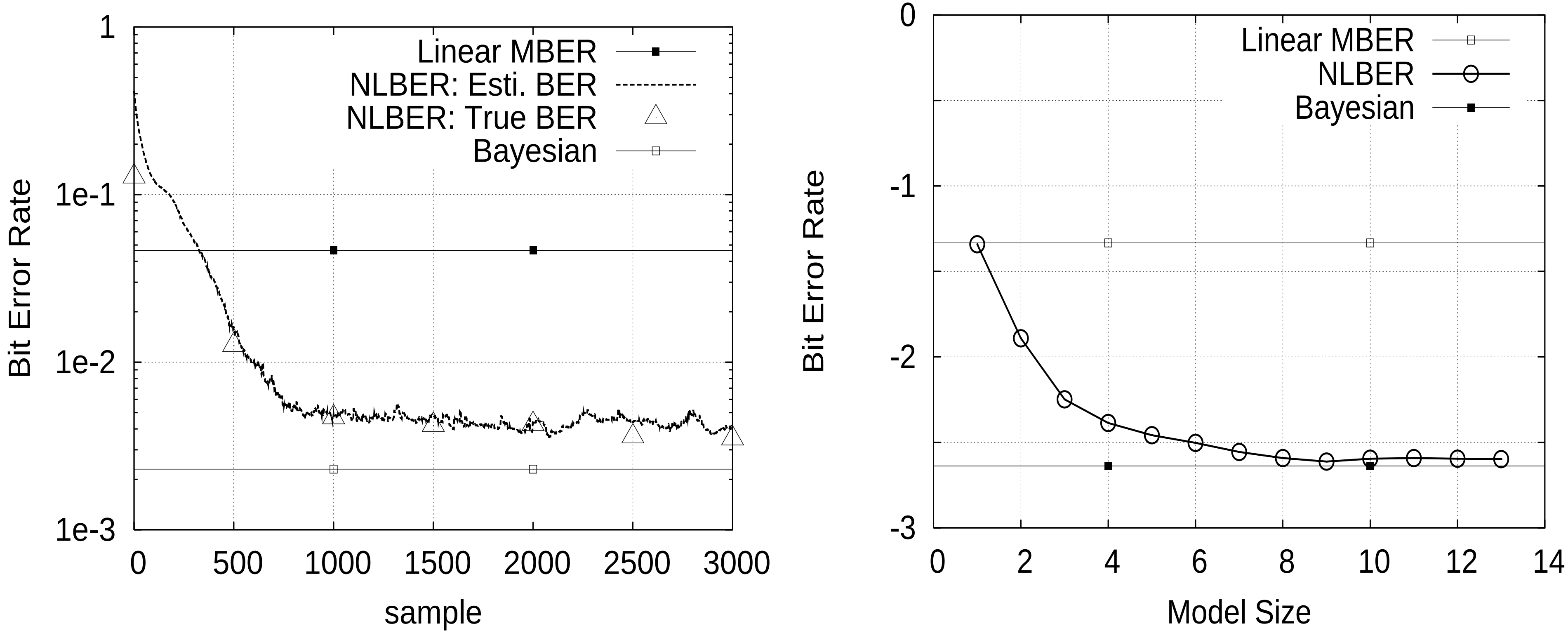
Desired-user's bit error rate



NLBER for Example One

(a) learning curve of **NLBER**-RBF detector with $N_c = 8$ averaged on 10 runs, and (b) influence of symmetric RBF **model size** N_c, where SNR$=7$ dB

(a) (b)



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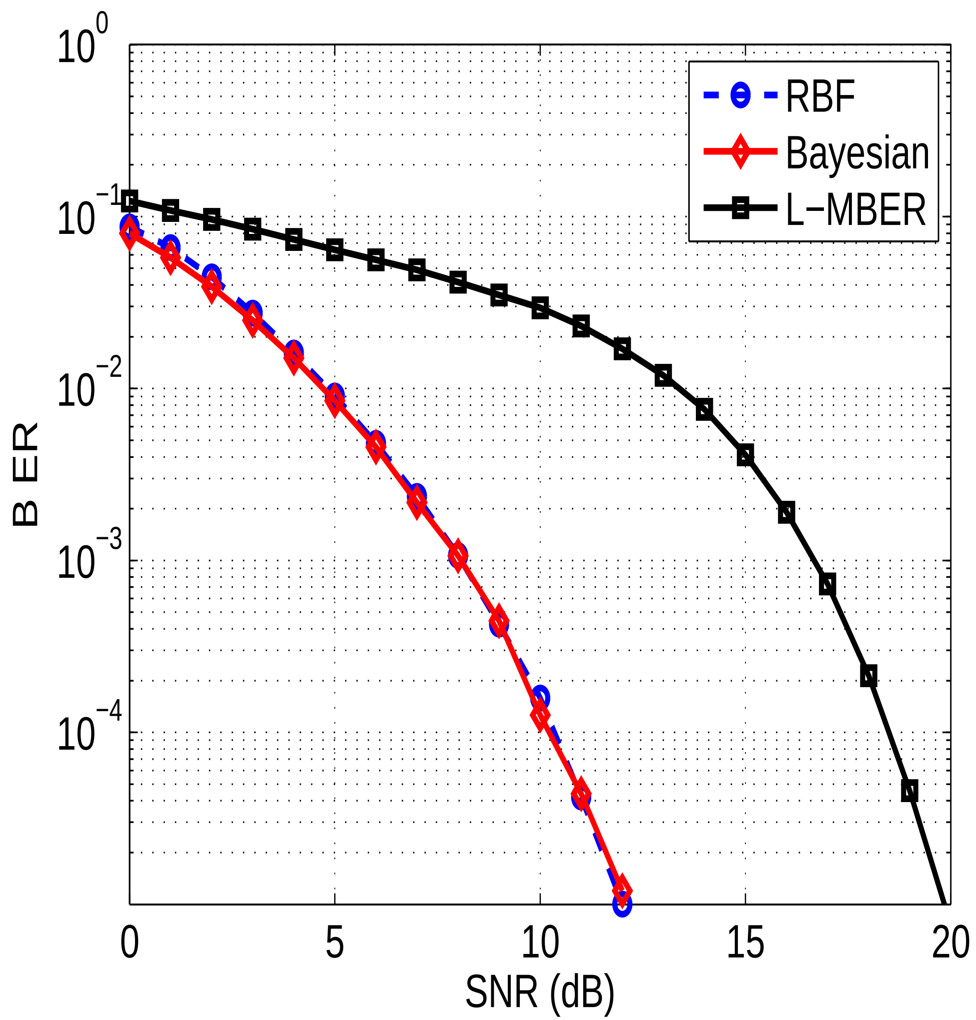
- **Symmetric** RBF network has $N_c = N_{sb} = 8$ symmetric RBF units
- **Clustering** algorithm, all RBF variances $\sigma_i^2 = \sigma_n^2$

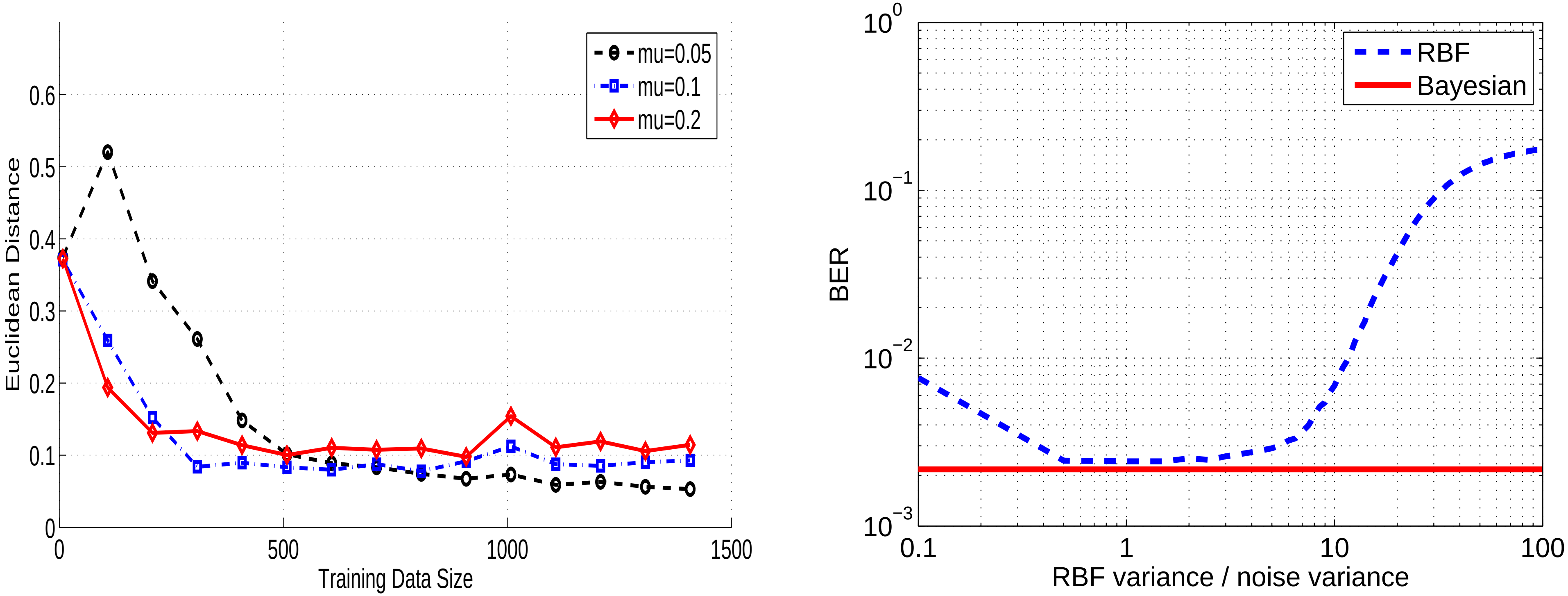
Desired-user's bit error rate



Clustering for Example One

(a) learning curve[1] of **clustering**-RBF detector with $N_c = 8$ averaged on 5 runs, and (b) influence of RBF **variance**, where SNR= 7 dB

(a) (b)



[1]Euclidean distance(k) is defined as sum of $\|\mathbf{c}_i(k) - \bar{\mathbf{x}}_i^{(+)}\|^2$ over $1 \le i \le N_{sb}$

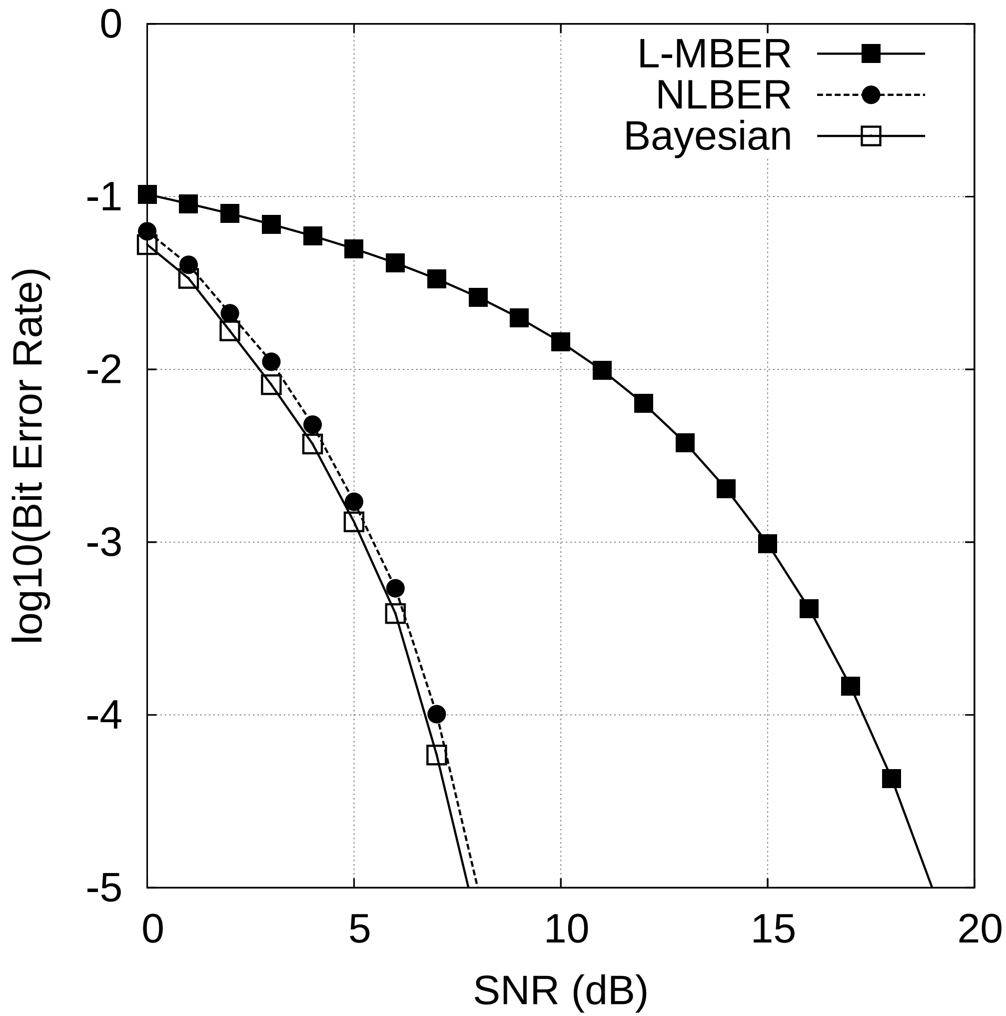
Simulation Example Two

- 3-element array with half wavelength spacing, five equal-power BPSK users
- Location of users in terms of **angle of arrival**

1	2	3	4	5
0°	10°	-17°	15°	20°

- **Symmetric** RBF network has $N_c = N_{sb} = 16$ symmetric RBF units
- **Nonlinear least bit error rate** algorithm

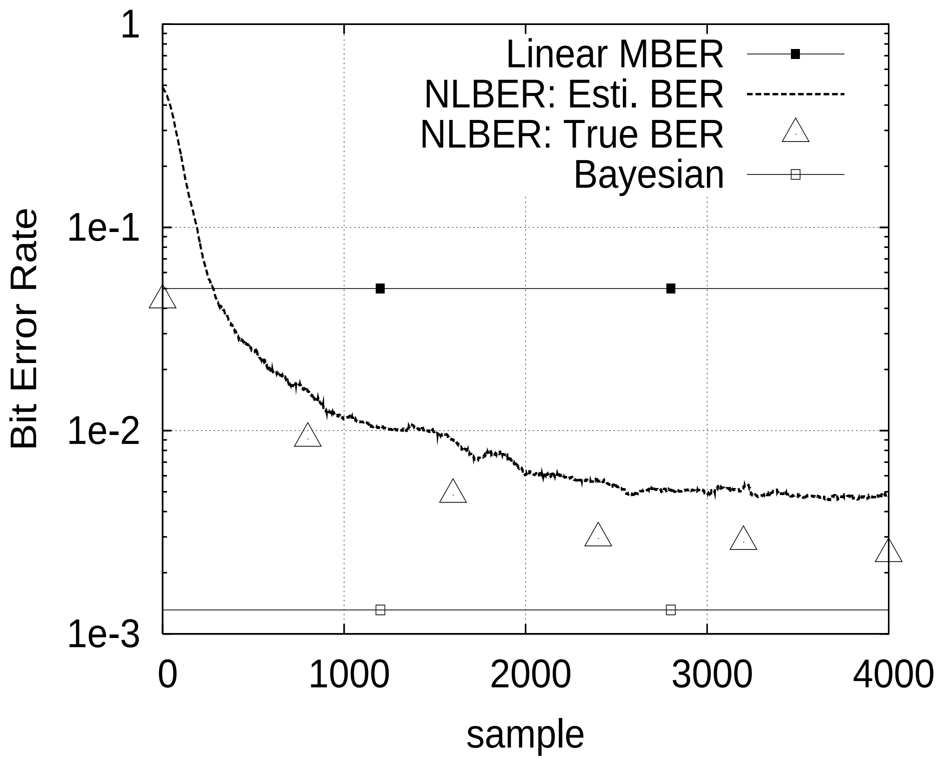
Desired-user's bit error rate



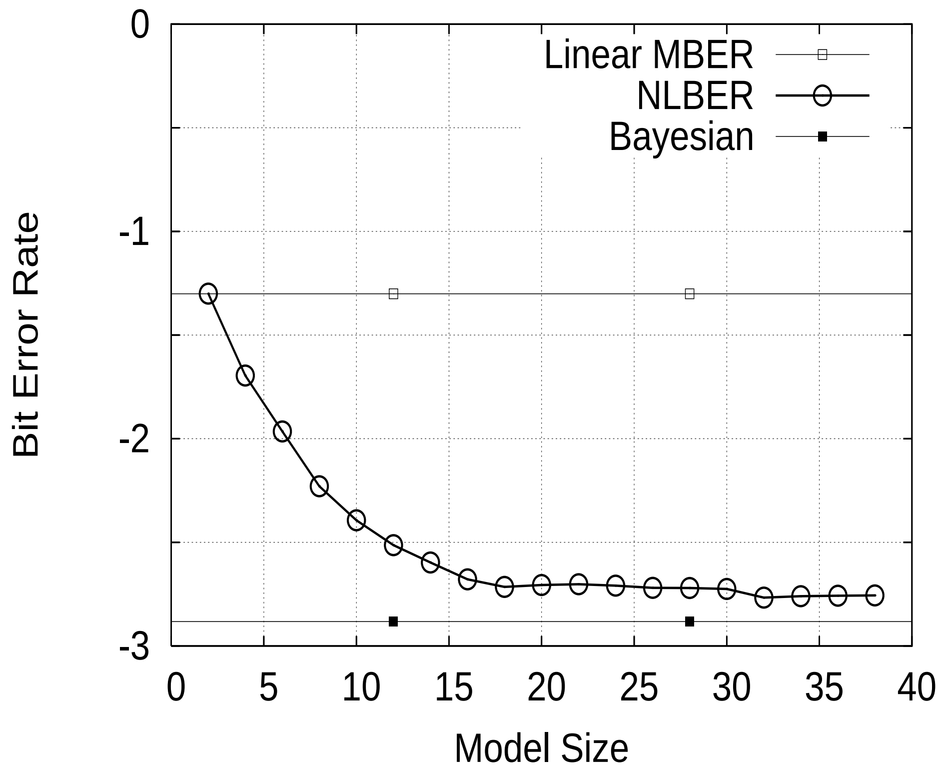
NLBER for Example Two

(a) learning curve of **NLBER**-RBF detector with $N_c = 16$ averaged on 10 runs, and (b) influence of symmetric RBF **model size** N_c, where SNR$= 5$ dB

(a)

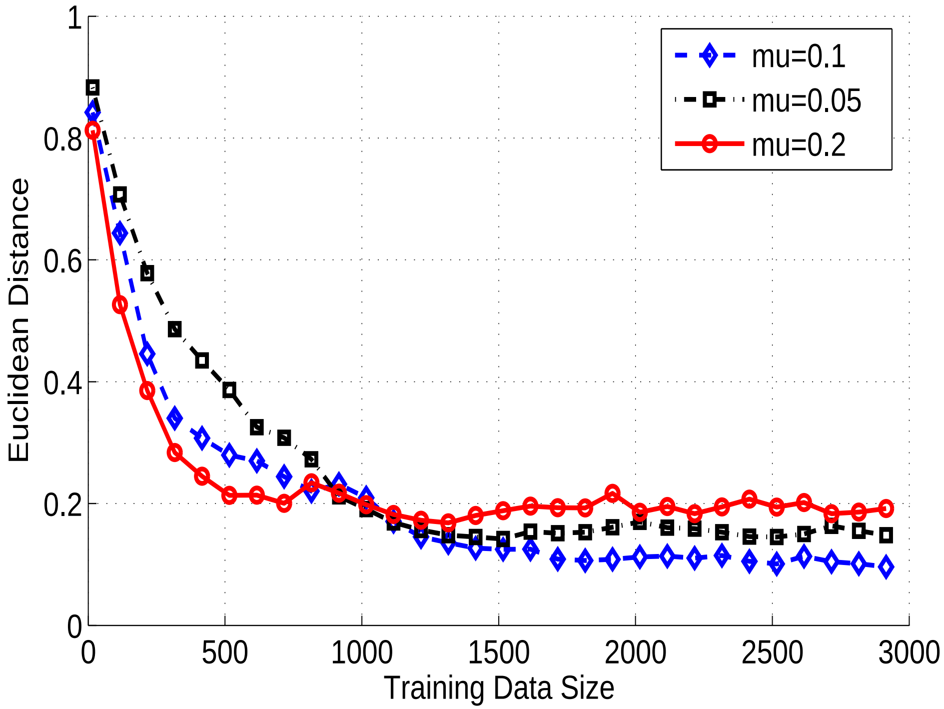


(b)



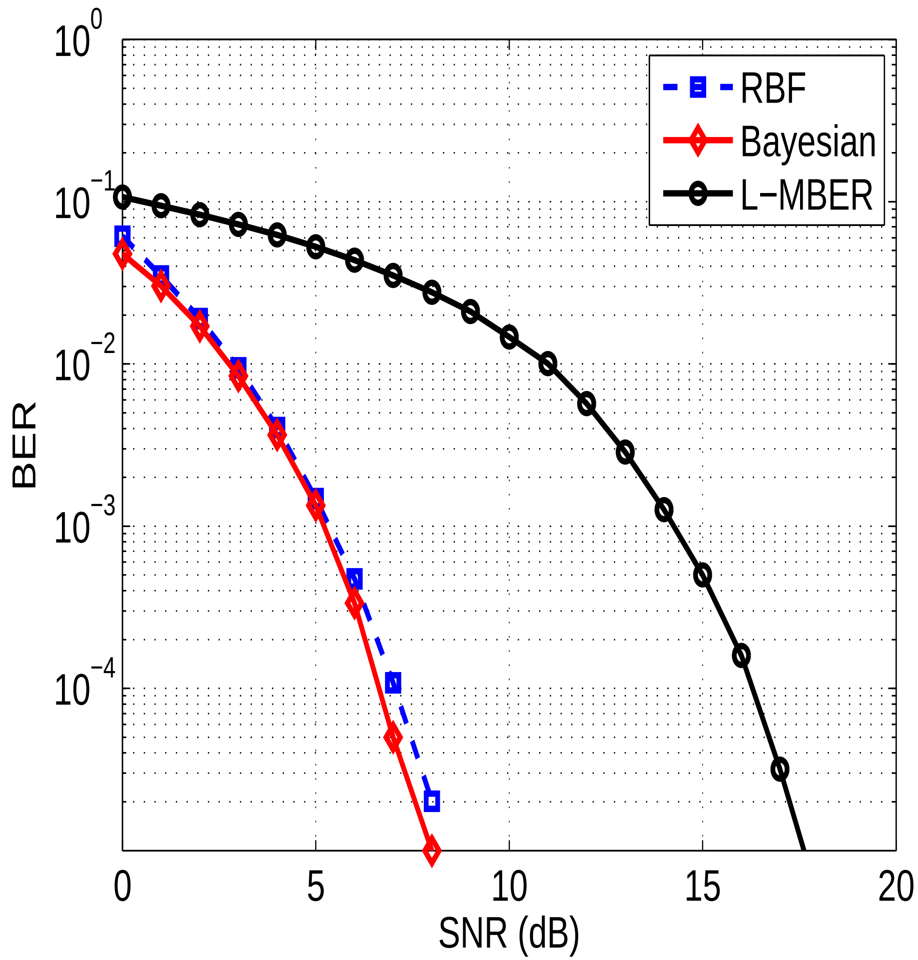
Simulation Example Two

Learning curve of **clustering**-RBF detector with $N_c = 16$ averaged on 5 runs, where SNR= 5 dB



Desired-user's bit error rate

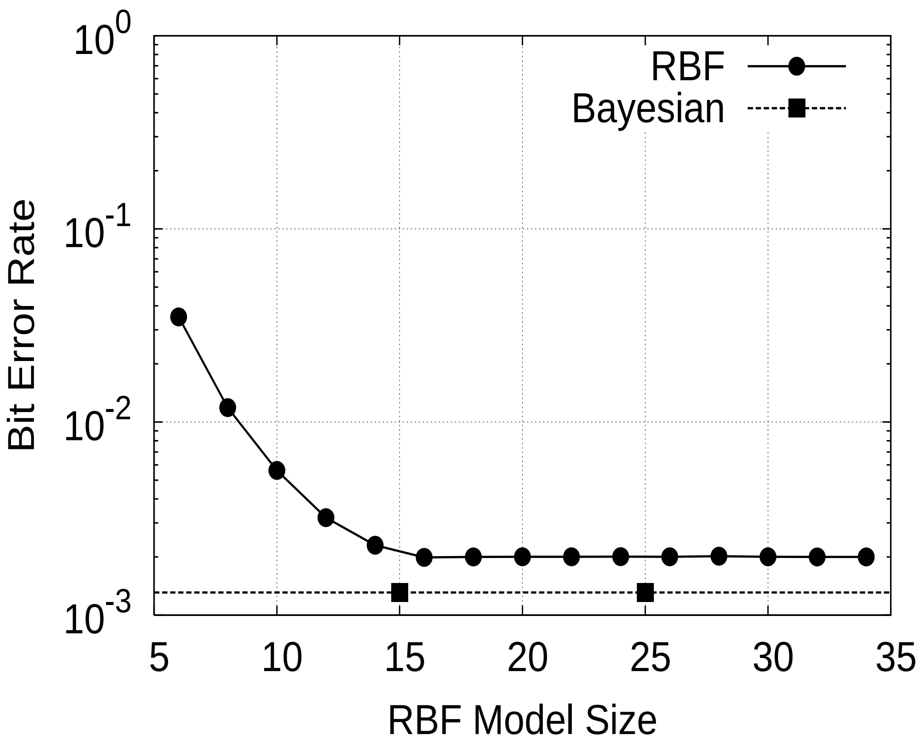
clustering-RBF detector $N_c = 16$



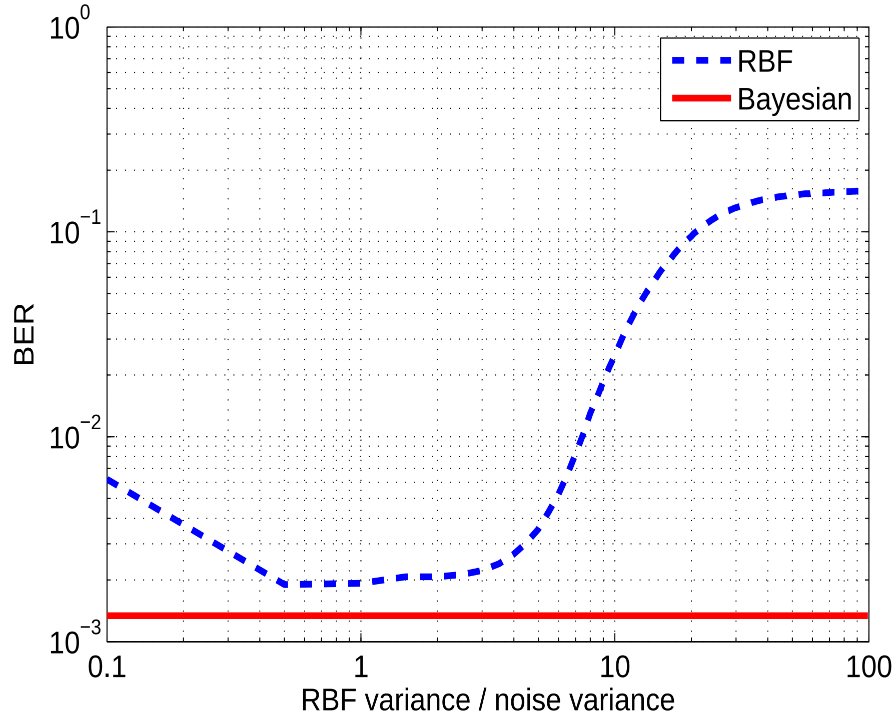
Clustering for Example Two

(a) influence of clustering-RBF **model size** N_c with RBF variance set to σ_n^2, and (b) influence of RBF **variance** with $N_c = 16$, where SNR$= 5$ dB

(a)



(b)



Conclusions

- ❏ Nonlinear beamforming achieves significantly smaller system bit error rate and larger user capacity
- ❏ Optimal Bayesian beamforming solution has an inherent symmetry structure
- ❏ A novel symmetric radial basis function network has been proposed for nonlinear beamforming
- ❏ Two adaptive algorithms for downlink senario
  - ☆ Nonlinear least bit error rate
  - ☆ Cluster-variation enhanced clustering

THANK YOU.

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