EXIT Chart Analysis of Low-Complexity Bayesian Turbo Multiuser Detection for Rank-Deficient Multiple Antenna Aided OFDM

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***Abstract*— This paper studies the mutual information transfer characteristics of a novel low-complexity Bayesian Multiuser Detector (MUD) proposed for employment in Space Division Multiple Access (SDMA) aided Orthogonal Frequency Division Multiplexing (OFDM) systems. The design of the Bayesian MUD advocated is based on extending the optimum single-user Bayesian design to multiuser OFDM signals modeled by a Gaussian mixture, rather than by a single Gaussian distribution, when characterizing the conditional PDF of the received signal. In order to reduce the complexity of the Bayesian MUD, we introduce an *a priori* information threshold and then discard the low-probability terms during the calculation of the extrinisic information generated . The achievable complexity reduction as a function of different threhold values is analyzed and the best tradeoff values are derived with the aid of simulation. Both non-systematic and recursive systematic convolutional codes are used for exchanging extrinsic information with the MUD for the sake of achieving a turbo-detection aided iteration gain. The convergence behavior of the proposed low-complexity Bayesian turbo MUD is investigated using EXtrinsic Information Transfer (EXIT) chart analysis and compared to that of Soft Interference Cancellation aided Minimum Mean Square Error (SIC-MMSE) MUD schemes. As expected, the simulation results show that the proposed low-complexity Bayesian Turbo MUD outperforms the SIC-MMSE MUDs. A substantial benefit of the proposed MUD is that it is potentially capable of supporting up to three times higher number of users than the number of receiver antennas. In this challenging multiuser scenario, the resultant channel-matrix becomes rank-deficient, resulting in a linearly non-separable detector output phasor constellation, when classic linear receivers tend to exhibit a poor performance.**

I. Introduction

During the past a few years, Space Division Multiple Access (SDMA) has attracted substantial research efforts expended for the sake of increasing the number of users supported. This is achieved by allowing the users to communicate within the same time-slot and frequency band, differentiating them with the aid of their unique user-specific Channel Impulse Response (CIR) [1], [2]. In recent years, Orthogonal Frequency Division Multiplexing (OFDM) has found its way into a range of Wireless Local Area Network (WLAN) and Broadcast standards. Combining the benefits of SDMA and OFDM has the promise of achieving reliable wireless communications at high data rates with the aid of efficient Multi-User Detector (MUD) algorithms [2].

Following Berrou's landmark paper on the turbo principle [3], iterative detection has found applications in channel coding [4], channel estimation, equalization [4] and multiuser detection [4]. The conventional Minimum Mean Square Error (MMSE) algorithms can be conveniently combined with the so-called interference Cancellation (IC) [2], [4] technique to create attractive MUDs. However, when supporting a higher number of users than the number of receiver antennas, the channel matrix becomes rank-deficient and hence the MMSE SDMA MUD falters. In this rank-deficient scenario the phasors become linearly non-separable at the channel's output and only non-linear SDMA MUDs have the ability to perform adequately. Hence in this contribution we propose a non-linear Bayesian turbo MUD, which is capable of achieving an attractive tradeoff between the attainable complexity reduction and the performance degradation imposed. Our advocated solution outperforms the linear turbo MMSE MUD benchmarkers at the cost of a moderate complexity increase [5].

The achievable performance of these MUDs will be investigated using EXIT charts [6], which provide a convenient way of visualizing the mutual information exchange between the inputs and outputs of concatenated receiver components and hence allow us to predict their achievable performance and to examine their convergence properties. Although at a low number of interferes the extrinsic information at the output of the proposed turbo Bayesian MUD may be non-Gaussian distributed, nonetheless, the EXIT-chat technique succeeded in predicting the achievable turbo performance of the system in the context of SDMA OFDM using convolutional codes and BPSK modulation.

Perfect Channel State Information (CSI) is assumed to be available at the receiver, which will be substituted by a more realistic channel estimator outlined for example in Chapter 16 of [2] in our future work. Alternatively, a number of adaptive techniques may be used for circumventing this problem [7]. Our results will demonstrate that the proposed detector is capable of significantly outperforming the MMSE MUD, especially in the so-called rank-deficient scenarios, namely when the number of SDMA uplink users supported is higher than the number of receiver antennas at the base station.

The remainder of this contribution is organized as follows. In Section II, a system model is introduced, which will be used in Section III for studying a range of different turbo MUD strategies. Our system performance results and EXIT chart analysis are presented in Section IV, followed by our conclusions in Section V.

II. System Model

The SDMA uplink (UL) transmission structure is portayed in Fig.1. More specifically, each of the L simultaneous MSs employs a convolutional encoder and a single UL transmission antenna, while the BS's UL receiver has a P-element antenna array. As seen in Fig. 1, the set of complex-valued UL signals, $x_p[n,k], p \in 1,\ldots,P$ received from the P-element antenna array in the k-th subcarrier of the n-th OFDM symbol is constituted by the superposition of the independently faded signals corresponding to the L UL users sharing the same frequency band, which are also corrupted by the Additive White Gaussian Noise (AWGN) encountered at the array elements [2]. The indices $[n,k]$ have been omitted for notational convenience during our forthcoming discourse, yielding [2]:

$$\mathbf{x} \quad = \quad \mathbf{Hs} + \mathbf{n} \; = \; \bar{\mathbf{x}} + \mathbf{n}, \qquad (1)$$

where $\mathbf{x}$ is the $(P\times1)$-dimensional vector of the UL received signals, $\mathbf{s}$ is the transmitted $(L\times1)$-dimensional signal vector generated from the convolutional encoded bits, $\mathbf{n}$ is the $(P\times1)$-dimensional noise vector and $\bar{\mathbf{x}}$ represents the noiseless component of $\mathbf{x}$. Both the complex-valued UL transmitted signal, s_l of the l-th user, where $l \in 1,\ldots,L$ and the AWGN process, n_p, at p-th antenna array element, where $p \in 1,\ldots,P$ are assumed to exhibit a zero mean and variances of σ_l^2 and $2\sigma_n^2$, respectively.

Furthermore, $\mathbf{H}$ is the Frequency Domain Channel Transfer Function (FDCHTF) matrix having a dimension of $(P\times L)$, constituted

The financial support of the European Union and that of EPSRC, UK is gratefully acknowledged.

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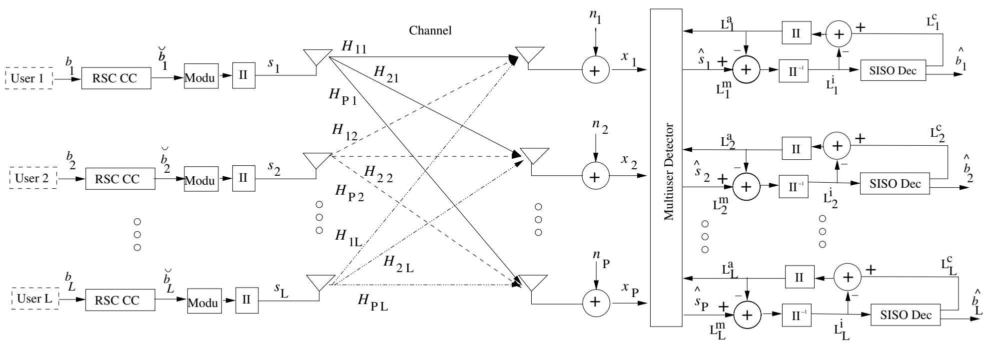

Fig. 1. Schematic of the turbo SDMA OFDM uplink scenario, where each of the L users is equipped with a convolutional channel code and a single transmit antenna, while the BS's receiver is assisted by a P-element antenna array followed by iterative processing.

by the set of channel transfer factors $H_{p,l}$, which describes the independent, stationary and complex-Gaussian distributed fading process between the reception array element $p \in 1, \ldots, P$ and the single transmitter antenna associated with a particular user l, characterized by a zero-mean and unit variance.

The *a posteriori* information $\mathbf{L}^{m,p}(s_l)$ output by the turbo MUD is derived by exploiting both the received signal vector **x** and the *a priori* information $\mathbf{L}^{m,a}(s_l)$ of all the L UL users, which is the interleaved extrinsic information $\mathbf{L}^{c,e}(s_l)$ generated by the channel decoders for the bits received. Upon subtracting the *a priori* information $\mathbf{L}^{m,a}(s_l)$ from the *a posteriori* information $\mathbf{L}^{m,p}(s_l)$, the extrinsic information $\mathbf{L}^{m,e}(s_l)$ output by the MUD is attained. After de-interleaving $\mathbf{L}^{m,e}(s_l)$ is forwarded to a bank of Soft-Input Soft-Output (SISO) channel decoders as the *a priori* information $\mathbf{L}^{c,a}(s_l)$, in order to generate the *a posteriori* information $\mathbf{L}^{c,p}(s_l)$ for carrying out the decisions concerning the original source bits. The extrinsic information $\mathbf{L}^{c,e}(s_l)$ output by the l-th channel decoders, where $l \in 1, \ldots, L$, is generated by deducting the *a priori* information $\mathbf{L}^{c,a}(s_l)$ from the *a posteriori* output $\mathbf{L}^{c,p}(s_l)$ and then interleaved, before it is fed back to the MUD as the *a priori* information $\mathbf{L}^{m,a}(s_l)$, in order to complete a full iteration. The subscript l of the soft information $\mathbf{L}$ in Fig.1 indicates that it belongs to s_l. The usual interleavers and de-interleavers separating the MUD and the channel decoders render the distribution of information fed into the MUD and channel decoders independent of each other, which improves the benefits of the extrinsic information generated in terms of the achievable iteration gain.

When employing BPSK modulation, the *a posteriori* information of the l-th user's transmitted bit s_l generated by the SISO MUD is expressed in terms of Log Likelihood Ratios (LLR) as:

$$\begin{aligned} \mathbf{L}^{m,p}(s_l) &= \mathbf{ln}\frac{P(s_l=+1|\hat{s}_l)}{P(s_l=-1|\hat{s}_l)} \\ &= \mathbf{ln}\frac{P(\hat{s}_l|s_l=+1)}{P(\hat{s}_l|s_l=-1)} + \mathbf{ln}\frac{P(s_l=+1)}{P(s_l=-1)} \\ &= \mathbf{L}^{m,e}(s_l) + \mathbf{L}^{m,a}(s_l), \end{aligned} \quad (2)$$

where the *a posteriori* information seen in Fig. 1 is given by the sum of the extrinsic information and *a priori* information. More explicitly, $\mathbf{L}^{m,e}(s_l)$ in Eq.(2) is the extrinsic information which is fed into the channel decoder of Fig. 1 after de-interleaving, while the second term denoted by $\mathbf{L}^{m,a}(s_l)$ represents the *a priori* information related to the interleaved encoded bits s_l. Since no *a priori* information is available for the MUD during the first iteration, we have $\mathbf{L}^{m,a}(s_l) = 0$. During the following iterations, the *a priori* information of the MUD is generated by interleaving the extrinsic information of the channel decoder, gained by subtracting its *a priori* information $\mathbf{L}^{c,a}(s_l)$ from its *a posteriori* information $\mathbf{L}^{c,p}(s_l)$, as depicted in Fig.1.

III. TURBO MUDS

In this section, several turbo MUDs will be introduced. The conventional MMSE MUD minimizing the complex-valued MSE between the estimated and ideal noiseless received signal will be presented first, which employs soft interference cancellation. This is referred as the Complex-valued MMSE (CMMSE) [5] MUD. For BPSK scenarios, the so-called real-valued MMSE (RMMSE) [8] may also be applied for enhancing the achievable performance of the system, which will be outlined below. Finally, we will investigate the novel low-complexity Bayesian MUD and demonstrate that it outperforms the linear benchmark employed.

For the linear turbo MUDs employing soft interference cancellation, such as the CMMSE and RMMSE schemes, the symbol $\bar{\hat{s}}^{(i)}_{l(MMSE)}$ of the l-th user estimated during the i-th decoding iteration can be written as follows [5]:

$$\hat{s}_l^{[i]} = \bar{s}_l^{[i]} + v_l^{[i]} \mathbf{w}_l^{[i]H} \cdot (\mathbf{x} - \mathbf{H}\bar{\mathbf{s}}^{[i]}), \quad (3)$$

where the superscript $[i]$ denotes i-th decoding iteration. $v_l^{[i]}$ is the *a priori* variance of $\hat{s}_l^{[i]}$ given by [5]:

$$v_l^{[i]} = 1 - |\bar{s}_l^{[i]}|^2, \quad (4)$$

$\bar{s}_l^{[i]}$ is the l-th user's *a priori* mean value and all the values $\bar{s}_l^{[i]}$, $l = 1 \ldots L$ of the L users constitute the mean vector $\bar{\mathbf{s}}^{[i]}$ of i-th decoding iteration. Furthermore, $\mathbf{w}_l^{[i]H}$ is the Hermitian of the l-th column of the array weight matrix $\mathbf{W}^{[i]}$ employing the CMMSE or the RMMSE criterion. For notational simplicity, we have omitted the superscript $[i]$ from $\bar{s}_l$, v_l, $\mathbf{w}_l$ and $\mathbf{W}$ during our forthcoming discourse.

A. Turbo CMMSE Multiuser Detection

The CMMSE MUD is one of the most popular linear MUD algorithms, which minimizes the Complex-valued MSE (CMSE) metric for the l-th user expressed as:

$$\mathbf{J}_{CMSE}(\mathbf{w}_l) = E[|\hat{s}_l^{[i]} - s_l|^2], \quad (5)$$

where $\hat{s}_l^{[i]}$ and s_l are generally complex-valued.

The closed-form CMMSE array weight vector is expressed as [5]:

$$\mathbf{w}_{l,CMMSE} = (\mathbf{HVH}^H + 2\sigma_n^2 \mathbf{I}_P)^{-1}\mathbf{h}_l \quad (6)$$

assuming that $\sigma_l^2 = 1$, where $\mathbf{V} = diag[v_1 \ldots v_L]$, $\mathbf{I}_P$ is a $(P \times P)$-dimensional identity matrix, and $\mathbf{h}_l$ is the l-th column of the FDCHTF matrix $\mathbf{H}$.

B. Turbo RMMSE Multiuser Detection

For BPSK transmission, only the real part of the estimated signal is required, therefore directly minimizing the real-valued MSE (RMSE) between the estimated signal and desired signal will remove the unnecessary constraint imposed on the array weight matrix of the CMMSE MUD characterized in Eq.6 and hence will provide significant performance improvements. The cost function of the RMMSE algorithms is formulated as [8]:

$$\mathbf{J}_{RMSE}(\mathbf{w}_l) = E[|\hat{s}_{l,R}^{[i]} - s_l|^2], \quad (7)$$

where $\hat{s}_{l,R}^{[i]}$is the real part of the estimated signal $\hat{s}_l^{[i]}$ during the i-th decoding iteration.

Applying the real-valued vertical concatenation matrix method of [8], we attain the vertically concatenated RMMSE weight vector in the form of: assuming that $\sigma_l^2 = 1$, where $\mathbf{h}_l$ is the l-th column of the FDCHTF matrix $\mathbf{H}$ and the subscript c of the matrices indicates the vertical concatenation, defined as:

$$\underset{\sim}{\mathrm{U}}_c = \begin{pmatrix} \Re[\underset{\sim}{\mathrm{U}}] \\ \Im[\underset{\sim}{\mathrm{U}}] \end{pmatrix}, \quad (8)$$

with $\underset{\sim}{\mathrm{U}}$ indicating an arbitrary matrix. While $\Re[\underset{\sim}{\mathrm{U}}]$ and $\Im[\underset{\sim}{\mathrm{U}}]$ denote real and imaginary parts, respectively. Once the vertically concatenated RMMSE weight vector $\mathbf{w}_{l,RMMSE,c}$ has been derived, the RMMSE weight vector $\mathbf{w}_{l,RMMSE}$ can be derived by finding the inverse of the operator in Eq.(8).

C. low-complexity Bayesian turbo Multiuser Detection

When the *a priori* information concerning the likelihood of all the legitimate $N_b = 2^L$ number of BPSK modulated L-user bit sequences becomes available, the joint PDF of the antenna array's output $\mathbf{x}$ and the transmitted BPSK modulated bits $\breve{b}_l^{(j)} \in \{\pm 1\}, j \in 1, \ldots, N_b$ of user l at the output of the convolutional encoder can be expressed as the superposition of all the conditional Gaussian PDFs positioned at the legitimate noiseless outputs corresponding to $s_l = +1$ and $s_l = -1$, multiplied by the j-th legitimate signal vector's probabilities, respectively, which can be expressed as :

$$P(\mathbf{x}, s_l = +1) = \sum_{\forall j: s_l^{(j)} = +1} P(s^{(j)})\ e^{\left(-\frac{(\|\mathbf{x} - \bar{\mathbf{x}}_j\|)^2}{2\sigma_n^2}\right)}, \quad (9)$$

$$P(\mathbf{x}, s_l = -1) = \sum_{\forall j: s_l^{(j)} = -1} P(s^{(j)})\ e^{\left(-\frac{(\|\mathbf{x} - \bar{\mathbf{x}}_j\|)^2}{2\sigma_n^2}\right)}. \quad (10)$$

The entire set of $N_b = 2^L$ number of legitimate vectors of the L users is partitioned into two subset corresponding to $s_l = +1$ and $s_l = -1$ according to these two equations. In Eq.9 and Eq.10, $2\sigma_n^2$ is the variance of the noise, while $\bar{\mathbf{x}}_j,\ j \in 1, \ldots, N_b$ constitutes the noiseless received signal vectors, where the Gaussian PDFs seen in Eq.9 and Eq.10 are centered. The Euclidian distance measured from their noiseless center to the received signal vector is used as the metric of quantifying their corresponding probability. Furthermore, $P(s^{(j)})$ is the a-priori information to be defined more explicitly below. All the other notations are defined as before.

Provided that the convolutional encoded bits of all the L users are independent, the probability $P(s^{(j)})$ can be expressed as :

$$P(s^{(j)}) = \prod_{l=1}^{L} P(s_l^{(j)}), j \in 1, \ldots, N_b, \quad (11)$$

where $P(s_l^{(j)})$ represents the probabilities of either $P(s_l^{(j)} = 1)$ or $P(s_l^{(j)} = 0)$, depending on the l-th user's bit at the j-th bit position, $j \in 1, \ldots, N_b = 2^L$ within the L-user transmitted symbol vector, which is the a-priori information provided by the l-th user's SISO channel decoder.

Based on our above discourse concerning the joint PDF of the received signal vector and the lth user's transmitted bit, we will ouline here the philosophy of the Bayesian turbo MUD advocated. Let us use the conditional likelihood of the received signal vector as that of the estimated bit decision concerning the lth user's transmitted bit according to Eq.2. Then we have:

$$\begin{aligned} \mathbf{L}^{m,p}(s_l) &= \ln \frac{P(s_l = +1|\mathbf{x})}{P(s_l = -1|\mathbf{x})} \\ &= \ln \frac{P(\mathbf{x}, s_l = +1)}{P(\mathbf{x}, s_l = -1)} \\ &= \ln \frac{\sum_{\forall j: s_l^{(j)} = +1} P(s^{(j)})\ e^{\left(-\frac{(\|\mathbf{x} - \bar{\mathbf{x}}_j\|)^2}{2\sigma_n^2}\right)}}{\sum_{\forall j: s_l^{(j)} = -1} P(s^{(j)})\ e^{\left(-\frac{(\|\mathbf{x} - \bar{\mathbf{x}}_j\|)^2}{2\sigma_n^2}\right)}}. \end{aligned} \quad (12)$$

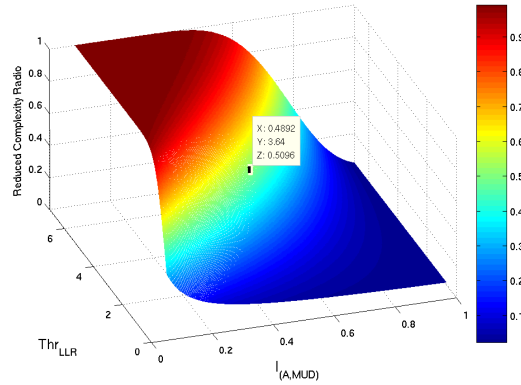

Fig. 2. Complexity reduction radio of the low-complexity Bayesian MUD when supporting $L = 6$ users

where all the notations have been define before. Eq.12 will lead to the maximum-likelihood solution.

In order to further reduce the complexity of the Bayesian turbo MUD, we introduce the *a priori* LLR threshold value Thr_{LLR}. Before invoking the *a priori* probability vector for calculating $\mathbf{L}^{m,p}(s_l)$, preprocessing of the *a priori* information is carried, as follows:

$$P(s_l = +1) = \begin{cases} 1 & \text{if } \mathbf{L}^{m,a}(s_l) > Thr_{LLR}; \\ 0 & \text{if } \mathbf{L}^{m,a}(s_l) < -Thr_{LLR}; \\ \frac{1}{2}(tanh(\frac{\mathbf{L}^{m,a}(s_l)}{2}) + 1) & \text{else}, \end{cases} \quad (13)$$

where $l \in 1, \ldots, L$.

Let us now observe Eq.12 again. Once the *a priori* information $\mathbf{L}^{m,a}(s_l)$ associated with a specific user l has an absolute value higher than Thr_{LLR}, half the number of the $2^{(L-1)}$ PDF terms in both the numerator and denominator will become zero, when calucating $\mathbf{L}^{m,a}(s_{l'}), l' \neq l$. Hence the total number of calculations required by all the L users is nearly halved. If the absolute value of the *a priori* information associated with another user is higher than Thr_{LLR} as well, the complexity imposed will be further halved. Hence the complexity associated with using this threshold can be expressed as:

$$\mathbf{C}_{LC} \approx \mathbf{C}_{Original} \cdot (1 - \frac{1}{2} P(|\mathbf{L}^{m,a}(s_l)| > Thr_{LLR}))^L, \quad (14)$$

where $\mathbf{C}_{Original}$ is the original complexity of the Bayesian MUD, which can be seperated into two parts, namely the calculation of the legitimate channel output states and the addition of the Gaussian PDFs . The computation of the legitimate channel output states requires $2 * 2^L P(2L - 1)$ operations, while the addition of the Gaussian PFDs requires $2^L(6P+1)$ operations, plus 2^L exp function evaluations, when considering a single transmitted bit. When the Gaussian distribution assumption is applied to the PDF of the LLRs, $\mathbf{L}^{m,a}(s_l)$ can be considered to be normally distributed with a mean of $\mu = s_l \cdot \sigma_{LLR}^2/2$ and a variance of σ_{LLR}^2 [6]. Then following a number of steps we can derive:

$$\begin{aligned} P(|\mathbf{L}^{m,a}(s_l)| > Thr_{LLR}) = Q\left[\frac{Thr_{LLR} - \frac{1}{2}\sigma_{LLR}^2}{\sigma_{LLR}}\right] \\ + Q\left[\frac{Thr_{LLR} + \frac{1}{2}\sigma_{LLR}^2}{\sigma_{LLR}}\right]. \end{aligned} \quad (15)$$

Furthermore, the *a priori* mutual information of $\mathbf{I}_{(A,MUD)}$ available for the MUD and calculated between the *a priori* information and the bipolar bits will be a function of a single parameter, namely that of the LLR variance of σ_{LLR}^2 [6],

System Parameters	
SDMA	
Number of users	2, 4, 5, 6 ,8
Number of receiver antennas	2
Channel impulse response	3-path SWATM symbol-invariant [2 p.78]
OFDM	
Number of subcarriers	128
Length of cyclic prefix	32
Modulation	BPSK
Channel Coding	
Type	NSC, RSC
Code rate	1/2
Constraint Length	4
Turbo interleaver block length	20480
Decoder type	Approximate Log MAP [4]

TABLE I
PARAMETERS FOR THE SIMULATIONS.

$$\mathbf{I}(\sigma_{LLR}) = 1 - \int_{-\infty}^{\infty} \frac{e^{\left(\frac{-(x-\sigma_{LLR}^2/2)^2}{2\sigma_{LLR}^2}\right)}}{\sqrt{2\pi}\sigma_{LLR}} \cdot \log_2\left(1+e^{-x}\right)dx. \quad (16)$$

Since the function $I(\sigma_{LLR})$ is monotonically increasing, it is invertible. Therefore, the complexity reduction ratio $\frac{\mathbf{C}_{LC}}{\mathbf{C}_{Original}}$ will be a function of the LLR threshold Thr_{LLR} and that of the *a priori* mutual information $\mathbf{I}_{(A,MUD)}$ of the MUD. Fig.2 portrays the 3-D complexity reduction ratio, when supporting $L = 6$ users. As expected, this ratio is a monotonically increasing function of the threshold Thr_{LLR}, while a monotonically decreasing function of the *a priori* mutual information $\mathbf{I}_{(A,MUD)}$ of the MUD. This implies that we have to set the value of Thr_{LLR} as low as possible without inflicting any significant performance degradation in order to obtain the best possible complexity reduction. The simulated results of Section IV will provide us with further insight on the best tradeoff value of Thr_{LLR} between maintaining a low complexity and imposing a performance degradation.

IV. SIMULATION RESULTS

In this section, the attainable performance of the proposed low-complexity Bayesian turbo MUD is investigated in comparison to the other two linear turbo MMSE MUDs outlined in Section III. The EXIT chart technique is employed as our semi-analytical convergence testing tool. Our system parameters are summarized in Table I. Each user has a different random interleaver. We will demonstrate that the low-complexity Bayesian turbo detector outperforms the two linear turbo MMSE MUDs employed as benchmarkers.

In Fig.3, we plot both the associated EXIT charts along with the associated simulated detection trajectories of the low-complexity Bayesian turbo receiver associated with different threshold values, when supporting $L = 6$ users, and employing a half-rate, constraint-length-4 Non-Systematic Convolutional (NSC) encoder for each UL user. The lines characterize the EXIT curves of the proposed low-complexity Bayesian turbo detector in conjunction with different threshold values, indicating that the system's performance is nearly remain virtually unimpaired, when the threshold values are higher than $Thr_{LLR} = 4$. The simulated detection trajectories in conjunction with different threshold values are also shown in Fig.3 using lines marked with arrows. This also indicates that the best tradeoff value of the LLR threshold is around $Thr_{LLR} = 4$. The iterative detection process commences from the origin of Fig.3, which represents the absence of a-priori information for the MUD. Then the decoding trajectory traverses to the MUD's EXIT curve in Fig.3, indicating that valuable LLR information was generated by the MUD. The resultant extrinsic information is forwarded to the channel decoders

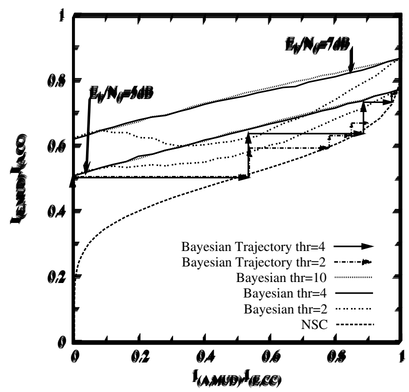

Fig. 3. EXIT charts and simulated trajectories of the low-complexity Bayesian MUD with different threshold values for $P = 2$ receiver antennas and $L = 6$ users

of Fig.1 and hence the detection trajectory reaches the EXIT curve of the channel decoder, which demonstrates that further extrinsic information is obtained from the SISO channel decoder assisted by the a-priori information gleaned from the MUD. During its further evolution, the trajectory traces up to the MUD's EXIT curve again, as a result of exploiting the extrinsic information extracted from the channel decoder, which is fed back to the MUD and so forth. In other words, the trajectory evolves in this manner within the open detection tunnel between the EXIT curves of the MUD and the channel decoder, until it reaches the intersection of the curves if the threshold is large enough. The simulated detection trajectories when the threshold equal to 4 or larger closely follow the behaviour predicted by the EXIT curves.

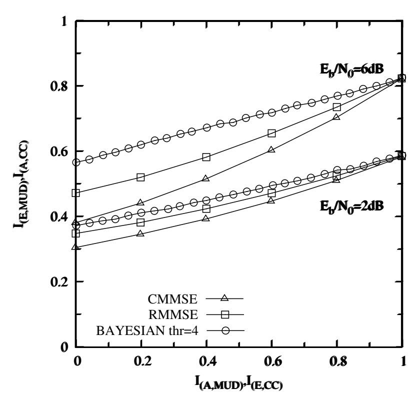

Fig. 4. EXIT charts of different turbo MUDs for $P = 2$ receiver antennas and $L = 6$ users

In order to plot the EXIT curves for the iterative multi-user communication system in a 2-D plane, the average of all the users' mutual information was used. If the average Channel Impulse Response (CIR) and SNR of each user is similar, then this may be judiciously exploited for the sake of employing EXIT analysis.

In Fig.4, we plot the EXIT curves of employing different turbo MUD schemes, including the CMMSE, the RMMSE and the proposed low-complexity Bayesian detectors with $Thr_{LLR} = 4$, when supporting $L = 6$ users at $E_b/N_0 = 2dB$ and $6dB$. At the abscissa of unity, all the EXIT curves of the different MUDs recorded at a given SNR value converge to the same point, coinciding with that produced by the MUD for a single user. This reveals that all of the three MUDs succeeded in completely eliminating the multiple access interference, when perfect a-priori information was provided. The Bayesian detector exhibits the highest ordinate value in Fig.4

at the abscissa of zero, followed by the RMMSE and then the CMMSE scheme, providing the widest EXIT tunnel. Hence the Bayesian detector has the lowest SNR convergence threshold and fastest convergence rate, again, followed by the RMMSE and the CMMSE detector. Fig.5 portrays the BER performance of these three

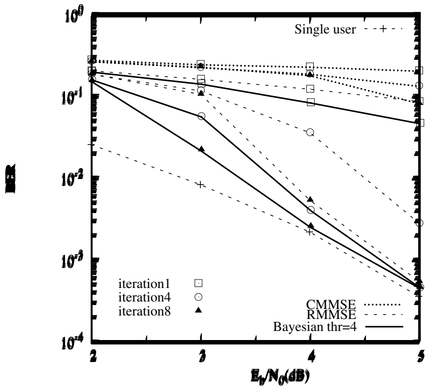

Fig. 5. BER performance of the different MUDs for $P = 2$ receiver antennas and $L = 6$ users

MUD algorithms parameterized by the SNR, which confirms the predictions of our EXIT chart analysis. It is clearly seen in Fig.4 that when the SNR is higher than a certain threshold and the number of iterations is sufficiently high, the achievable BER of each of the three MUDs approaches the single-user-bound.

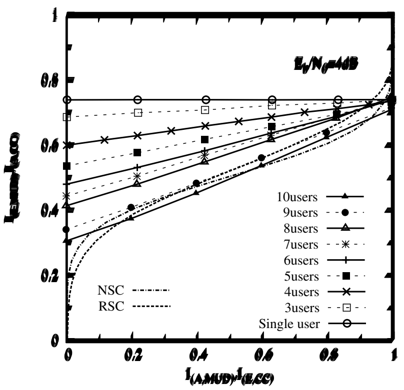

Fig. 6. EXIT characteristics of the NSC and RSC channel decoders and that of the low-complexity Bayesian MUD with $thr = 4$ for supporting different number of users at $E_b/N_0 = 4dB$, when employing $P = 2$ receiver antennas

Fig.6 allows us to compare the EXIT curves of different convolutional channel codes, namely that of the previously used NSC code as well as that of the half-rate, constraint-length-4 Recursive Systematic Convolutional (RSC) code to the corresponding EXIT curve of the Bayesian MUD, when supporting different number of users at $E_b/N_0 = 4dB$. We can see in Fig.6 that all the curves corresponding to the different numbers of users converge to the same point, again, attaining a near-single-user performance. This indicates that regardless of the number of users, all the multiple access interference can be removed when perfect a-priori information is available, namely, at the abscissa value of unity. The differences of these curves supporting different numbers of users may be observed both in terms of their different ordinate values at abscissa value of zero and the sloping rates. The higher the number of users supported, the lower ordinate starting points in Fig.6 and the steeper their slopes. This is because supporting a higher number of users imposes more interference at the MUD's output, resulting in a lower extrinsic mutual information improvement, when the same amount of a-priori information is provided. More explicitly, Fig.6 suggests that at $E_b/N_0 = 4dB$, at least $L = 8$ users can be supported, since there is an open EXIT-tunnel between the MUD's and the channel decoder's EXIT curves. Let us briefly compare the different channel codes adopted, namely the NSC and RSC code having the generator polynomials of (15,17) and (13,6) expressed in terms of their octal representation, respectively. We may conclude from Fig.6 that the shapes of the EXIT curves associated with the NSC and RSC are similar, although the NSC code has the potential of offering the benefits of a slightly wider EXIT tunnel initially, at the cost of having a lower ordinate value at the abscissa of zero. This implies having a higher residual BER for the NSC code.

The best matching channel decoder for the MUD considered can be found for example by using the Irregular Convolutional Codes (IRCC) derived in [9].More explicitly, this procedure allows us to shape the IRCC scheme's EXIT characteristic so that it matches that of the MUD and hence provides an open EXIT-tunnel at low SNRs, thereafter it is capable of potentially supporting near-capacity operation.

V. Conclusion

In conclusion, a novel low-complexity Bayesian turbo MUD was proposed, which is capable of outperforming both the CMMSE and the RMMSE turbo MUDs. We also demonstrated the accuracy of EXIT charts, despite the non-Gaussian LLR-distribution recorded at the output of the Bayesian MUD. The focus of our future work is on the design of similar turbo receivers for high-throughput QAM schemes using a variety of sophisticated channel decoders [4].

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